

# Covered Material and Resources for Part1 (0 - Exam 1)

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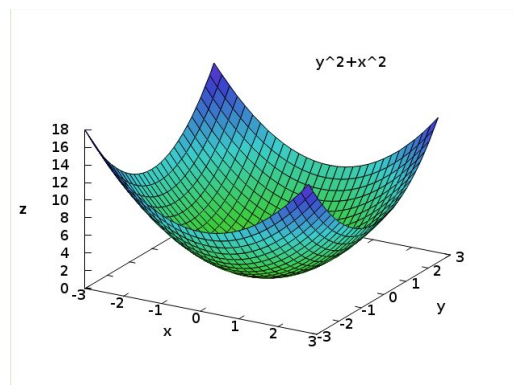
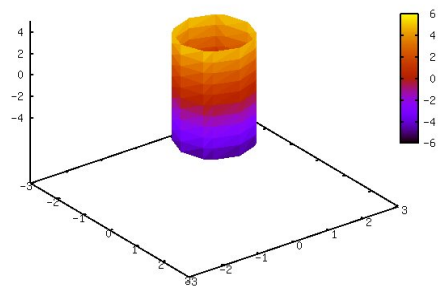
## 1 August 25

Textbook: Covered section 10.1 - (3d-coordinate system), introduced section 10.2 (vectors)

Covered 3d-coordinate system, points, distance formula, plots of surfaces ( $x^2 + y^2 = r^2$ ) and multivariable functions (e.g.  $z = f(x, y) = x^2 + y^2$ ). Equation of sphere. Examples with distance from point to plane, distance to axis, equation of plane equidistant between two points. Other references:

- Vector Calculus by Corral (p.1-10) <http://www.mecmath.net/calc3book.pdf>
- <http://tutorial.math.lamar.edu/Classes/CalcIII/3DCoords.aspx>
- <http://tutorial.math.lamar.edu/Classes/CalcIII/QuadricSurfaces.aspx>
- <http://tutorial.math.lamar.edu/Classes/CalcIII/MultiVrbleFcns.aspx>
- [www.mathcentre.ac.uk/resources/uploaded/mc-ty-introvector-2009-1.pdf](http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-introvector-2009-1.pdf)

Note the difference between graph of quadric surface  $x^2 + y^2 = 1$ , with  $z \in \mathbb{R}$  and of the function  $z = f(x, y) = x^2 + y^2$ :



## 2 August 27

Textbook: Covered section 10.2 - vectors, operations with vectors, components of vectors, unit vectors, parallel vectors

Introduced basic operations with vectors (addition, subtraction, and scalar multiplication), standard basis vectors, writing vectors in terms of standard basis vectors, force problems (tug of war, cable tension, etc). Examples with distance between spheres, testing if points are on a line, vector component problems, riverboat with current problem.

Other references:

- Vector Calculus by Corral (p.1-15) <http://www.mecmath.net/calc3book.pdf>
- Wikipedia: [http://en.wikipedia.org/wiki/Vector\\_addition#Addition\\_and\\_subtraction](http://en.wikipedia.org/wiki/Vector_addition#Addition_and_subtraction)
- Paul's notes" <http://tutorial.math.lamar.edu/Classes/CalcII/VectorArithmetic.aspx>
- tug of war: <https://www.khanacademy.org/math/precalculus/vectors-precalc/magnitude-direction/v/adding-three-vectors>
- vectors on youtube: <https://www.youtube.com/watch?v=1L8NzLjNAWI>
- riverboat: <http://www.physicsclassroom.com/class/vectors/Lesson-1/Relative-Velocity-and-Riverboat-Problems>

## 3 August 29

Textbook: Covered section 10.3 - dot product and its applications including angle, projections, and work

law of cosines, dot product formula, examples with find angle between vectors, find unit vector orthogonal to both  $\hat{i} + \hat{j}$  and  $\hat{i} + \hat{k}$ , angles in diagonals of a cube, signed scalar projection component and vector projection of vector  $b$  on  $a$ , when  $\theta < \frac{\pi}{2}$  and  $\frac{\pi}{2} < \theta < \pi$ . Work done by constant force moving object in given direction.

Other references:

- Vector Calculus by Corral (p.9-27): <http://www.mecmath.net/calc3book.pdf>
- Paul's notes: <http://tutorial.math.lamar.edu/Classes/CalcII/DotProduct.aspx>
- law of cosines: <http://mathproofs.blogspot.com/2006/06/law-of-cosines.html>
- angle between vectors: [https://www.khanacademy.org/math/linear-algebra/vectors\\_and\\_spaces/dot\\_cross\\_products/v/defining-the-angle-between-vectors](https://www.khanacademy.org/math/linear-algebra/vectors_and_spaces/dot_cross_products/v/defining-the-angle-between-vectors)
- dot product and projections: <http://math.msu.edu/~gnagy/teaching/10-fall/mth234/w2-234-h.pdf>

- MIT opencourseware on vectors: <http://ocw.mit.edu/courses/mathematics/18-02sc-multivariable-calculus-fall-2010/1.-vectors-and-matrices/part-a-vectors-determinants-and-planes/session-1-vectors/>

Look in Paul's notes for how to test for perpendicular and parallel vectors using vector dot product.

## 4 September 3

Textbook: Covered section 10.4 - cross product and its applications (area of parallelogram and volume of parallelepiped, torque to be discussed)

Result of cross product is a vector (unlike result of dot product),  $2 \times 2$  and  $3 \times 3$  determinants, test for parallel vectors, areas and volumes

Other references:

- Vector Calculus by Corral: <http://www.mecmath.net/calc3book.pdf>
- Paul's notes: <http://tutorial.math.lamar.edu/Classes/CalcII/CrossProduct.aspx>
- Wikipedia: [http://en.wikipedia.org/wiki/Cross\\_product](http://en.wikipedia.org/wiki/Cross_product) and <http://en.wikipedia.org/wiki/Parallelepiped>

## 5 September 5

Textbook: Finished section 10.4 - cross product and its applications (area of parallelogram, volume of parallelepiped, and torque), discussed section 10.5 - lines in 3D

Reviewed area of parallelogram and volume of parallelepiped using magnitude of cross and triple scalar product, respectively. Reviewed Cauchy-Schwarz and triangle inequality (these are fundamental inequalities). Discussed torque, another application of cross product: the tendency of force to rotate an object about an axis. Gave examples using force applied to door with hinges. Door is easier/harder to open or close (rotate around hinges) depending on where force is applied relative to position of hinges. Introduced lines in 3D - vector form, component form, symmetric form. Find line through two points, find intersection point of two lines, find out if lines are parallel or skew.

Other references:

- Vector Calculus by Corral: <http://www.mecmath.net/calc3book.pdf>
- Paul's notes: <http://tutorial.math.lamar.edu/Classes/CalcIII/EqunsOfLines.aspx>
- Torque: <http://physics.bu.edu/~duffy/py105/Torque.html> and <http://patrickjmt.com/torque-an-application-of-the-cross-product/>

## 6 September 8

Textbook: Finished section 10.5 - lines and planes in 3D, started section 10.6 - 3D surfaces

Equations of lines and planes. Line through two points. Distance (shortest) from a point to the line. Point of intersection of two lines. Skew lines. Plane through 3 points. Angle between planes. Line of intersection of two planes (note that the direction vector of such a line is perpendicular to the normal vectors of both planes i.e.  $\vec{v} = \vec{n}_1 \times \vec{n}_2$ ). Find line through a point perpendicular to a plane. Find equation of plane passing through a point containing a given line. Find equation of plane through two points that is perpendicular to a given line. Plot paraboloids, hyperboloids of one sheet.

Other references:

- Vector Calculus by Corral: <http://www.mecmath.net/calc3book.pdf>
- Paul's notes on lines and planes: <http://tutorial.math.lamar.edu/Classes/CalcIII/EqnsOfLines.aspx> and <http://tutorial.math.lamar.edu/Classes/CalcIII/EqnsOfPlanes.aspx>
- Paul's notes on surfaces and graphs in 3D: <http://tutorial.math.lamar.edu/Classes/CalcIII/QuadricSurfaces.aspx> and <http://tutorial.math.lamar.edu/Classes/CalcIII/MultiVrbleFcns.aspx>.
- Various youtube videos on lines, planes, and surfaces: <https://www.youtube.com/watch?v=y4y5DvNovQo>, <https://www.youtube.com/watch?v=JAG1zsIt93U>, <https://www.youtube.com/watch?v=jozabh0lFmo>, <https://www.youtube.com/watch?v=UWbEd-9yfD0>

## 7 September 10

Textbook: Examples from 10.5 (lines and planes), 10.7 (vector valued functions and space curves)

Distance between point and a plane, distance between parallel planes, find equations of two parallel planes a certain distance away. Reflection of a point in a plane. Examples of vector valued functions: helix, line segment, circle. Intersection of cylinder with plane. Properties of vector valued functions (derivative of cross product, etc).

Other references:

- Vector Calculus by Corral: <http://www.mecmath.net/calc3book.pdf>
- Paul's notes on lines and planes: <http://tutorial.math.lamar.edu/Classes/CalcIII/EqnsOfLines.aspx> and <http://tutorial.math.lamar.edu/Classes/CalcIII/EqnsOfPlanes.aspx>
- Reflection of point in plane: <http://mathforum.org/library/drmath/view/54763.html>
- Distance between planes: <https://www.youtube.com/watch?v=ctah61AqXrQ>
- Paul's notes on vector valued functions: <http://tutorial.math.lamar.edu/Classes/CalcIII/VectorFunctions.aspx> and <http://tutorial.math.lamar.edu/Classes/CalcIII/VectorFcnsCalculus.aspx>

## 8 September 12

Textbook: 10.8 (Tangent Vectors, Arclength, Binomial Vector, Curvature)

Arclength and reparametrization. Computing coordinates of point after traveling a certain distance on a curve. Tangent, normal and binomial vectors. Computing curvature (three different formulas).

Other references:

- Paul's notes on vector valued functions: <http://tutorial.math.lamar.edu/Classes/CalcIII/VectorFunctions.aspx> and <http://tutorial.math.lamar.edu/Classes/CalcIII/VectorFcnsCalculus.aspx>
- Paul's notes on arclength and curvature: <http://tutorial.math.lamar.edu/Classes/CalcIII/VectorArcLength.aspx> and <http://tutorial.math.lamar.edu/Classes/CalcIII/Curvature.aspx>
- Paul's notes on tangent and normal vectors: <http://tutorial.math.lamar.edu/Classes/CalcIII/TangentNormalVectors.aspx>
- Youtube on TNB (tangent,normal,binomial): <https://www.youtube.com/watch?v=LNMUXQiRFxw>

## 9 September 15

Textbook: 10.8 and 10.9 (TNB, Curvature, Velocity and Acceleration)

Review of TNB and curvature. Different ways to compute curvature. Notice the difference between derivatives with respect to  $s$  (arc length) and  $t$  (time). Velocity and acceleration from  $r(t)$ . Tangential and normal components of acceleration. Note that  $a(t)$  has no component in direction of binomial vector  $B$  (since that is perpendicular to plane defined by  $T$  and  $N$ ). Note again that when time dependent vector  $A(t)$  has constant magnitude, that is  $\|A(t)\| = C$  for all time, then  $A(t) \cdot A'(t) = 0$  so  $A'(t)$  orthogonal to  $A(t)$  as long as  $\|A'(t)\| \neq 0$ . This is easy to see for uniform circular motion where  $v(t)$  is constant and  $a(t)$  is perpendicular to  $v(t)$  with magnitude  $\frac{v^2}{R}$ . Note that  $a(t)$ , which is a vector, need not be zero even if speed is constant, if direction of motion is changing. Note that two nonzero vectors  $u$  and  $v$  are parallel if their cross product is zero. We did an example in class where we checked if person 1 following a certain trajectory  $r(t)$  can see person 2 located at point  $P$ . For that, we must check if the vector  $r(t) - P$  is parallel at some  $t$  to  $v(t) = r'(t)$  which can be done by checking if their cross product is zero and solving for  $t$  where that occurs.

Other references:

- Vector Calculus by Corral (p. 54): <http://www.mecmath.net/calc3book.pdf>
- Paul's notes on arclength and curvature: <http://tutorial.math.lamar.edu/Classes/CalcIII/VectorArcLength.aspx> and <http://tutorial.math.lamar.edu/Classes/CalcIII/Curvature.aspx>

- Paul's notes on tangent and normal vectors: <http://tutorial.math.lamar.edu/Classes/CalcIII/TangentNormalVectors.aspx>
- Paul's notes on velocity and acceleration: [http://tutorial.math.lamar.edu/Classes/CalcII/Velocity\\_Acceleration.aspx](http://tutorial.math.lamar.edu/Classes/CalcII/Velocity_Acceleration.aspx)
- Illustration of TNB: <http://www.solitaryroad.com/c253.html>
- Uniform circular motion: <http://www.physicsclassroom.com/mmedia/circmot/ucm.cfm>

## 10 September 17 and 19

These dates corresponded to material in Section 11.1 and 11.2. However, we did several examples for the material from 10.1 – 10.9. In particular, how to find a normal plane at a given point of  $r(t)$ , how to find  $T$ ,  $N$ , and  $B$  when the normal plane at some point along the curve is perpendicular to a given plane, different ways to find curvature, find point on  $r(t)$  that is closest to a given point not on  $r(t)$  (using calculus).

## 11 Exam 1 Info and Suggestions

The Exam will cover sections 10.1 to 10.9 in the book. The exam will be similar to the previous exams so expect to get a problem involving vectors, lines and planes, tangent, normal, binomial vectors, curvature, and problems with path/velocity/arclength. All of these may involve dot and cross product operations and identifying parallel and perpendicular vectors. There may also be problems with quadric surfaces (know the basic shapes, how to plot them by identifying points on the surface, how to plot 2d slices of them) and something involving vector projections may also come up.

First, make sure you re-look at the textbook examples (Stewart). Alongside, you can look at web-resources mentioned in this pdf. These include the Vector Calc book by Corral, Paul's notes, etc. You may also want to view some of the following videos:

- Intersection of line and plane: <https://www.youtube.com/watch?v=qVvvy5hsQwk>
- Line of intersection of two planes: <https://www.youtube.com/watch?v=jozabh01Fmo>
- Distance between points, lines, and planes: <https://www.youtube.com/watch?v=x06Q1GVaChU>, <https://www.youtube.com/watch?v=v8plb6V8BQo>
- Arclength: <https://www.youtube.com/watch?v=5fP443JvTUg>

There is plenty of other useful videos on youtube. Once you are clear on these concepts, review your homework assignments (solutions are posted on the main course page) and old exam solutions. Try some of the old exam questions without checking the solution. Good luck.