

# Background overview and select applications.

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Math, engineering, and computer science education from U.S. and Internationally. Applied math, scientific and high performance computing, large scale data analysis background.

B.S. in Applied Mathematics (minor Comp. Science) from Engineering School.

M.A. in Applied and Computational Mathematics.

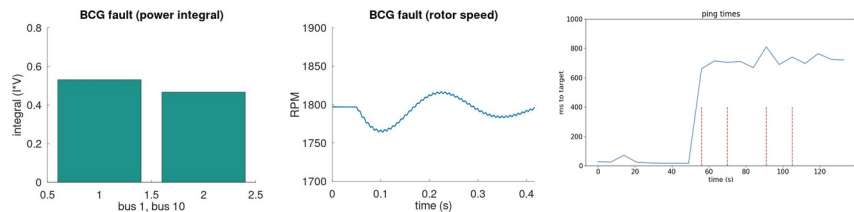
(Numerical differential equations, fluid mechanics and research in benchmarking general circulation climate models.)

Ph.D. in Applied and Computational Mathematics, Princeton Univ., 2012.

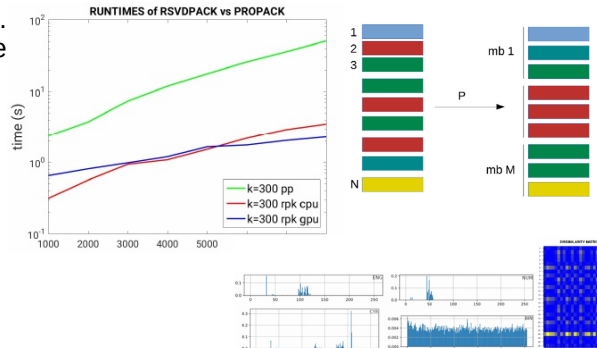
(Optimization problems with sparsity constraints, compressive sensing methods, applications to Geophysics and imaging, HPC implementations).

Work in academia post Ph.D. defense (2012 - 2017): signal processing, optimization, randomized algorithms for matrix manipulations / factorizations, imaging.

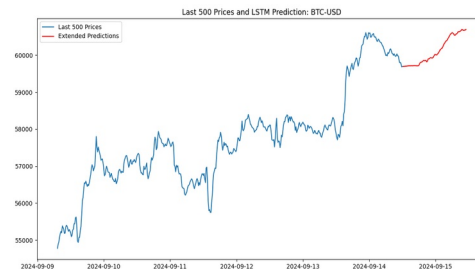
Work in industry (2017 - present): lossless compression, audio and video analysis / classification, anomaly detection / localization in electrical systems, multivariate time series, network analysis, etc. PI on multiple SBIR/STTR grants.



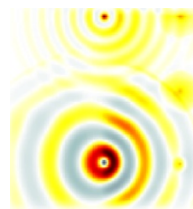
Development of statistical and AI-based methods for anomaly detection and localization in electrical systems and computer network applications.



Parallelized implementations with novel approaches for dimensionality reduction and lossless data compression.



Parameter optimized machine learning implementations for multivariate time series (e.g. financial price/vol data). LSTM based iterative predictions, Gaussian processes.

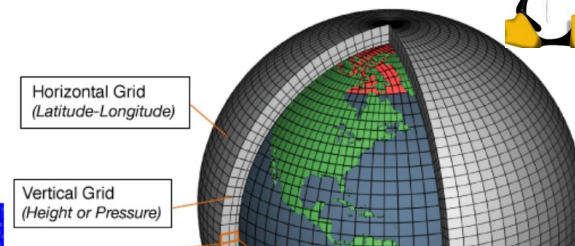


Signal processing algorithms and software for audio, imagery, video. Microphone and antenna array applications.

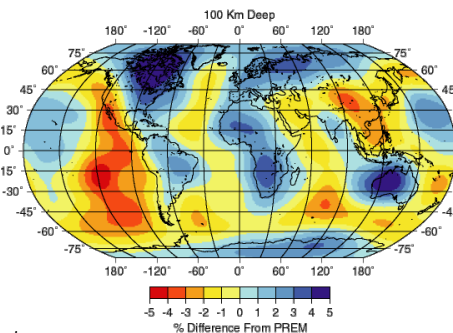


Development of approaches for high noise / blur image reconstruction.

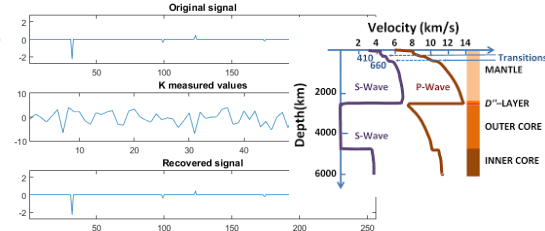
Different HPC and accelerated implementations with OpenMP, MPI, GPU.



Algorithms and software for Geotomographical inversion from seismic measurements.

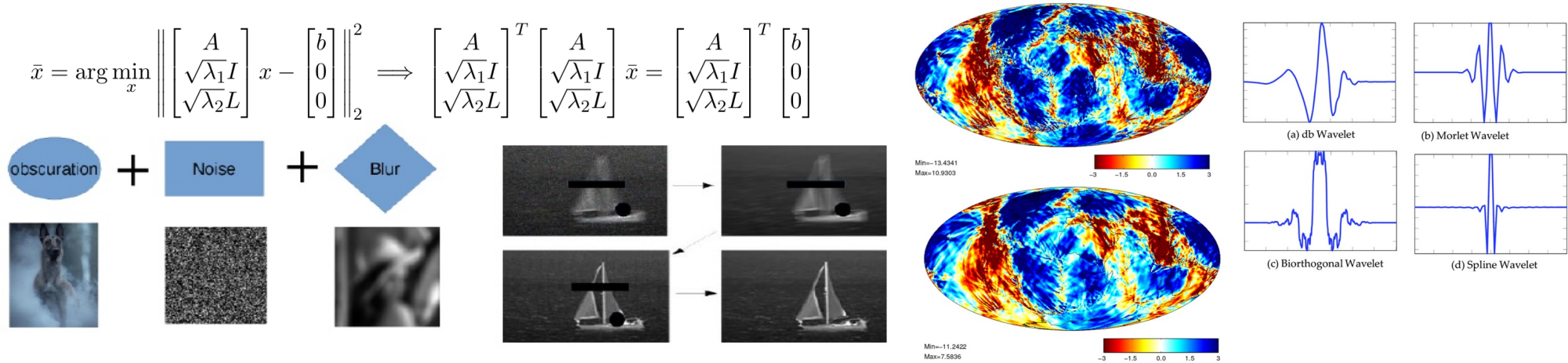


Compressed sensing developments. Sparse signal / transformed recovery.

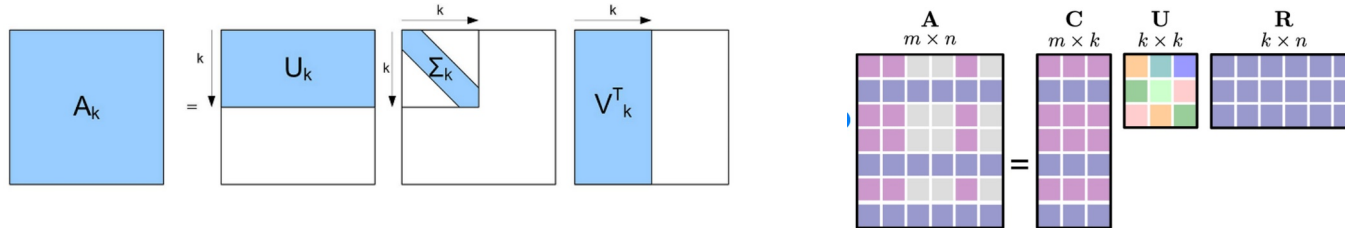


# Background

Postdoc (CNRS). Investigation of optimization based seismic inversion schemes for large data sizes on limited hardware. Developed projection and splitting methods. MPI implementation.



Postdoc (CU Boulder). Investigated randomized algorithms for obtaining low rank matrix factorizations (e.g. SVD, ID, CUR). Implemented RSVDPACK package. Instructor, N. Wiener Assistant Prof. (Tufts). Statistics, HPC.



Research / Sr. Scientist (Intelligent Automation, Inc.). Filed proposals and white papers to DOD/DOT/DOE. PI on different topics including data compression / multi-channel systems / waveform formation with antenna arrays. Contributor to projects on PTSD detection, aircraft trajectory analysis, interceptor models, multi-fidelity simulations, traffic management, etc.

Research Scientist (Intel Corp.). Network data collector, analyzer, anomaly detector. Multivariate time series predictors, sorting, compression.

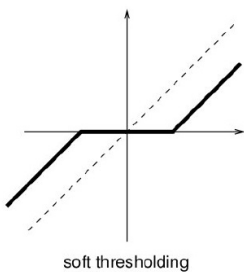
# Compressive sensing and sparsity constrained opt

Seek most efficient representation in some basis.

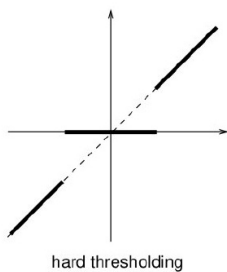
$$(I) Ax=b, \min \|x\|_0$$

$$(II) Ax=b, \min \|x\|_1 \rightarrow Ax \approx b, \min \|x\|_1 \rightarrow \min \|Ax - b\| + \lambda \|x\|_1$$

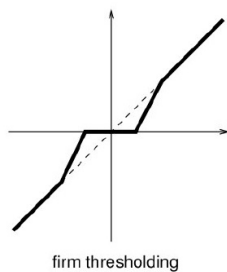
(I) and (II) equivalent under some conditions on A. (II) is a tractable problem. Much interest in minimization of  $\ell_1$  penalized functional.



soft thresholding



hard thresholding

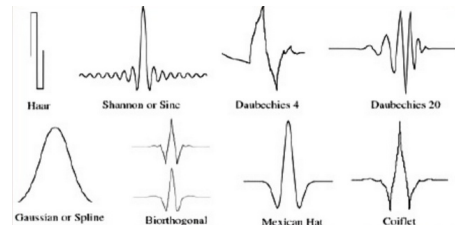
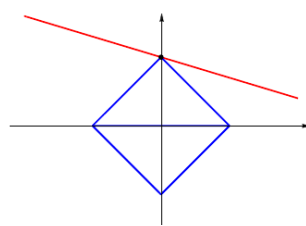


firm thresholding

$$y^0 = x^0, \quad t_1 = 1, \quad x^{n+1} = S_\tau(y^n - A^T(Ay^n - b))$$

$$t_{n+1} = \frac{1 + \sqrt{1 + 4t_n^2}}{2}$$

$$y^{n+1} = x^n + \frac{t_n - 1}{t_{n+1}}(x^n - x^{n-1})$$



$$\bar{w} = \arg \min_w \|RW^{-1}w - b\|_l + \lambda \|w\|_p \quad ; \quad x = W^{-1}\bar{w}$$

Simple iterative schemes depend on weighing factors and thresholding. BLAS 2/3 parallelization potential.

$$|x_k| = \frac{x_k^2}{|x_k|} = \frac{x_k^2}{\sqrt{x_k^2}} \approx \frac{x_k^2}{\sqrt{x_k^2 + \epsilon^2}}$$

$$x_k^{n+1} = \frac{1}{1 + \lambda_k q_k w_k^n} (x_k^n + (A^T b)_k - (A^T A x^n)_k) \quad \text{for } k = 1, \dots, N,$$

Generalized variable residual and solution norm scheme, involves multiple iterations of CG solves.

$$(A^T R^n A + (D^n)^T (D^n)) x^{n+1} = A^T R^n b$$



# Approximation / projection techniques for large data sizes

Wavelet thresholding and low rank projection methods.

$$A = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_m \end{bmatrix} \rightarrow M = \begin{bmatrix} \mathbb{T}(W r_1^T)^T \\ \mathbb{T}(W r_2^T)^T \\ \vdots \\ \mathbb{T}(W r_m^T)^T \end{bmatrix} = \mathbb{T}(A W^T) \approx A W^T$$

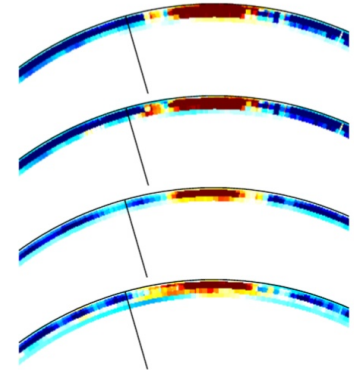
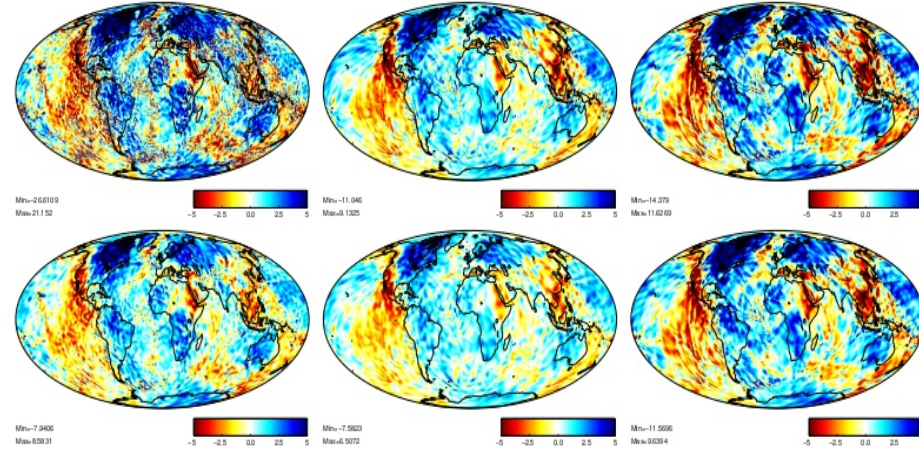
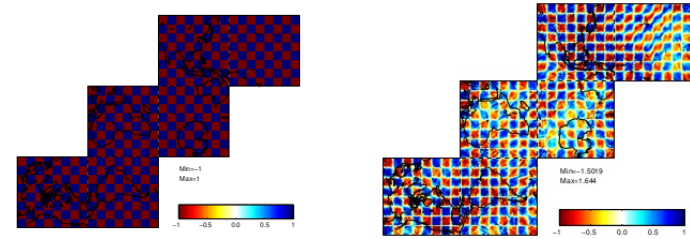
$$Mx \approx A W^T x \quad \text{and} \quad M^T y \approx (A W^T)^T y = W A^T y,$$

$$Ax \approx M W^{-T} x \quad \text{and} \quad A^T y \approx W^{-1} M^T y.$$

Projectors from first k eigenvectors.

$$\begin{bmatrix} U_{k_1}^T A_1 \\ U_{k_2}^T A_2 \\ \vdots \\ U_{k_p}^T A_p \end{bmatrix} x = \begin{bmatrix} U_{k_1}^T b_1 \\ U_{k_2}^T b_2 \\ \vdots \\ U_{k_p}^T b_p \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} \Sigma_{k_1} V_{k_1}^T \\ \Sigma_{k_2} V_{k_2}^T \\ \vdots \\ \Sigma_{k_p} V_{k_p}^T \end{bmatrix} x = \begin{bmatrix} U_{k_1}^T b_1 \\ U_{k_2}^T b_2 \\ \vdots \\ U_{k_p}^T b_p \end{bmatrix}$$

Decreased runtimes enabling more data to be used, but less detail at greater depths, where resolution is poor.

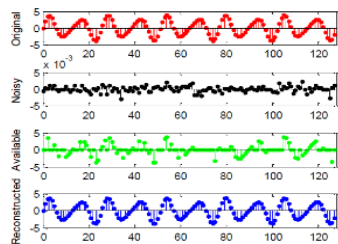
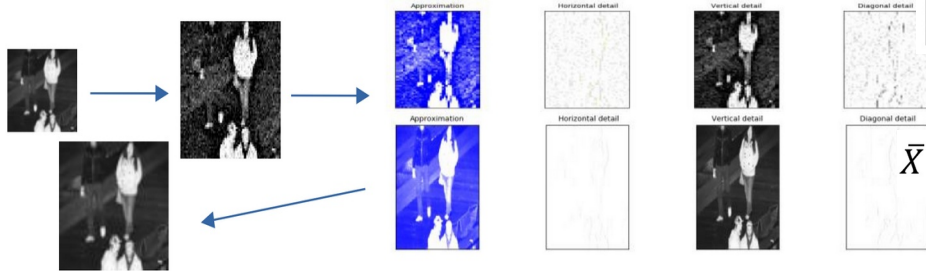


# Applications to image enhancement



$$W^{\{-1\}}(Thr(Wx)) \quad V(I) = \sum_{ij} \sqrt{|I_{i+1,j} - I_{i,j}|^2 + |I_{i,j+1} - I_{i,j}|^2}$$

## Image upscaling via CS + residual correction



Gradient based reconstruction of missing pixels [Stankovic et al].

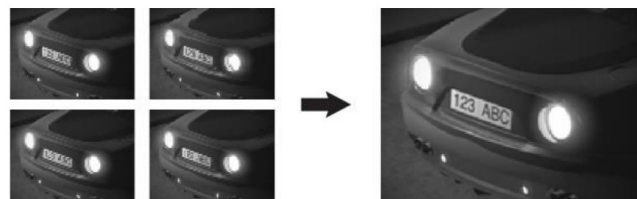
$$x_a^{(0)}(n) = \begin{cases} 0 & \text{for missing samples, } n \in \mathbb{N}_Q \\ x(n) & \text{for available samples, } n \in \mathbb{N}_A \end{cases} \quad \min \|X_a\|_1 \quad \text{subject to } x_a^{(m)}(n) = x(n) \quad \text{for } n \in \mathbb{N}_A$$

$$g(x, y) = h(x, y) \star f(x, y) + n(x, y),$$

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

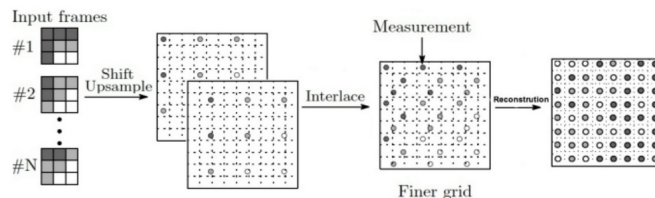
$$\hat{F}(u, v) = W(u, v)G(u, v)$$

$$W(u, v) = \frac{H^{\{*\}}(u, v)}{|H(u, v)|^2 + K(u, v)}$$



## Super-resolution

$$\bar{X} = \operatorname{argmin}_x \left\{ \sum_{k=1}^M \|D_k H_k F_k X - Y_k\|_2^2 + \lambda R(X) \right\}$$



CS based (e.g. matrix completion) pixel reconstruction

$$\mathcal{D}_\tau(\mathbf{X}) := \mathbf{U} \mathcal{D}_\tau(\Sigma) \mathbf{V}^*,$$

$$\mathbf{A} \mapsto \arg \min \frac{1}{2} \|\mathbf{X} - \mathbf{A}\|_F^2 + \lambda \|\mathbf{X}\|_*$$

$$\begin{aligned} &\text{minimize} && \|\mathbf{X}\|_* \\ &\text{subject to} && X_{ij} = M_{ij}, \quad (i, j) \in \Omega, \end{aligned}$$

Research combinations of transform / thresholding, optimization based and machine learning methods.

# Randomized algorithms

Choose large N,

> N = 1e5; x = randn(N,1); y = randn(N,1);

> x = x/norm(x); y = y/norm(y);

> abs(x\*y)

ans = 0.0033332

>

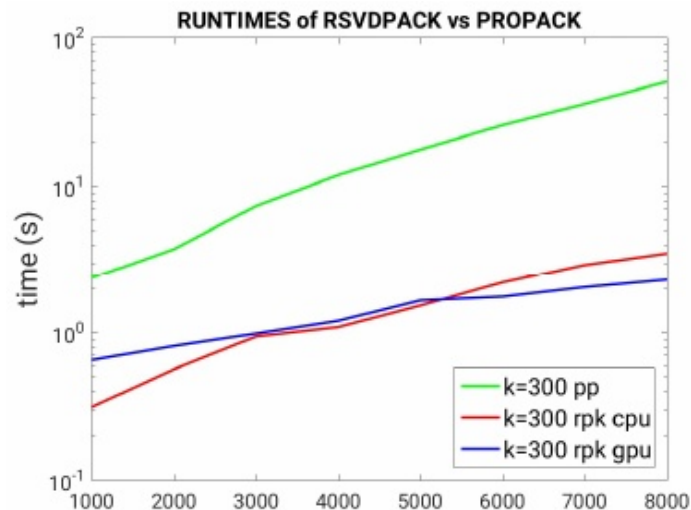
O(mnk) vs O(mn^2)

Sample range of  $A$  with  $k + p$  lin. indep. vectors, so that  $QQ^*A \approx A$ .

- Draw an  $n \times (k + p)$  Gaussian random matrix  $\Omega$ .  
 $\Omega = \text{randn}(n, k+p)$
- Form the  $m \times (k + p)$  sample matrix  $Y = A\Omega$ .  
 $Y = A * \Omega$  ;  $\text{ran}Y \approx \text{ran}A$
- Form an  $m \times (k + p)$  orthonormal matrix  $Q$  such that  $Y = QR$ .  
 $[Q, R] = \text{qr}(Y)$  ;  $\text{ran}Q \approx \text{ran}A$
- Form the  $(k + p) \times n$  matrix  $Q^*A$ .  
 $B = Q' * A$
- Compute the SVD of the smaller  $(k + p) \times n$  matrix  $B$ :  $B = \hat{U}\Sigma V^*$ .  
 $[\text{Uhat}, \text{Sigma}, V] = \text{svd}(B)$
- Form the matrix  $U = Q\hat{U}$ .  
 $U = Q * \text{Uhat}$  ;  $QQ^*A \approx A$
- $U_k = U(:, 1:k)$ ,  $\Sigma_k = \Sigma(1:k, 1:k)$ ,  $V_k = V(:, 1:k)$ .

$$A = [u_1 \dots u_r] \begin{bmatrix} \sigma_1 & 0 & \dots & 0 \\ 0 & \sigma_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_r \end{bmatrix} \begin{bmatrix} v_1^* \\ \vdots \\ v_r^* \end{bmatrix} = U\Sigma V^*$$

$$\approx U_k \Sigma_k V_k^* = [u_1 \dots u_k] \begin{bmatrix} \sigma_1 & 0 & \dots & 0 \\ 0 & \sigma_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_k \end{bmatrix} \begin{bmatrix} v_1^* \\ \vdots \\ v_k^* \end{bmatrix}$$



How to determine factorization rank adaptively based on tolerance? General form for different factorizations, parallelizable block based hierarchical approach for large sizes.

$$\|A - QB\| < \epsilon \Rightarrow QB = QQ^T A \Rightarrow QB \approx Q\hat{U}SV^T \quad \bar{y} = (I - QQ^T)y \quad \text{range}(\bar{Q}) = \text{span}(\text{range}(Q) \cup y).$$

$$q = \frac{\bar{y}}{\|\bar{y}\|} \quad \bar{Q}^T \bar{Q} = I_{r+1}.$$

$$\bar{Q} = [Q, q]$$

Instead of adding one vector at a time, add blocks at once.

**function**  $[Q, B] = \text{randQB\_pb}(M, \varepsilon, q, b_p)$

- (1) **for**  $i = 1, 2, 3, \dots$
- (2)  $\Omega_i = \text{randn}(n, b_p).$
- (3)  $Q_i = \text{orth}(M\Omega_i).$
- (4) **for**  $j = 1 : q$
- (5)  $Q_i = \text{orth}(M^T Q_i).$
- (6)  $Q_i = \text{orth}(MQ_i).$
- (7) **end for**
- (8)  $Q_i = \text{orth}(Q_i - \sum_{j=1}^{i-1} Q_j Q_j^T Q_i)$
- (9)  $B_i = Q_i^T M$
- (10)  $M = M - Q_i B_i$
- (11) **if**  $\|M\| < \varepsilon$  **then stop**
- (12) **end while**
- (13) Set  $Q = [Q_1 \cdots Q_i]$  and  $B = [B_1^T \cdots B_i^T]^T$ .

Hierarchical parallel implementation:

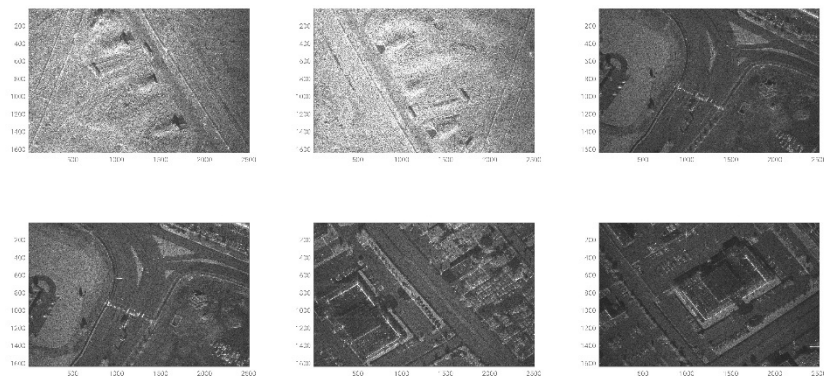
$$M = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{bmatrix} \approx \begin{bmatrix} Q_1 B_1 \\ Q_2 B_2 \\ Q_3 B_3 \\ Q_4 B_4 \end{bmatrix} = \begin{bmatrix} Q_1 & 0 & 0 & 0 \\ 0 & Q_2 & 0 & 0 \\ 0 & 0 & Q_3 & 0 \\ 0 & 0 & 0 & Q_4 \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix}$$

$$M^{(1)} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \approx Q_{12} B_{12} \quad ; \quad M^{(2)} = \begin{bmatrix} B_3 \\ B_4 \end{bmatrix} \approx Q_{34} B_{34} \quad M^{(3)} = \begin{bmatrix} B_{12} \\ B_{34} \end{bmatrix} \approx Q_{1234} B_{1234}$$

$$M \approx \begin{bmatrix} Q_1 & 0 & 0 & 0 \\ 0 & Q_2 & 0 & 0 \\ 0 & 0 & Q_3 & 0 \\ 0 & 0 & 0 & Q_4 \end{bmatrix} \begin{bmatrix} Q_{12} & 0 \\ 0 & Q_{34} \end{bmatrix} \begin{bmatrix} B_{12} \\ B_{34} \end{bmatrix} \approx \begin{bmatrix} Q_1 & 0 & 0 & 0 \\ 0 & Q_2 & 0 & 0 \\ 0 & 0 & Q_3 & 0 \\ 0 & 0 & 0 & Q_4 \end{bmatrix} \begin{bmatrix} Q_{12} & 0 \\ 0 & Q_{34} \end{bmatrix} Q_{1234} B_{1234}$$

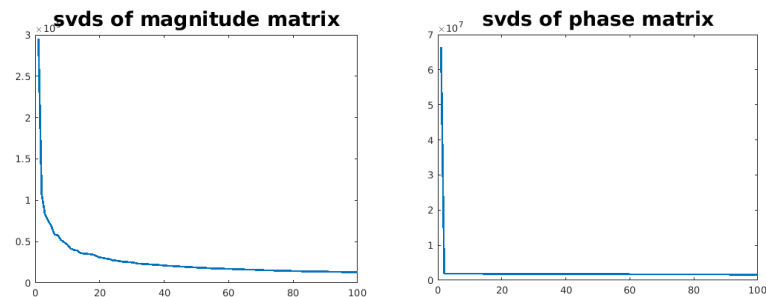
$$= Q^{(3)} Q^{(2)} Q^{(1)} B^{(1)} = QB$$

Many applications of low rank matrix / tensor factorizations ... e.g. complex SAR imagery



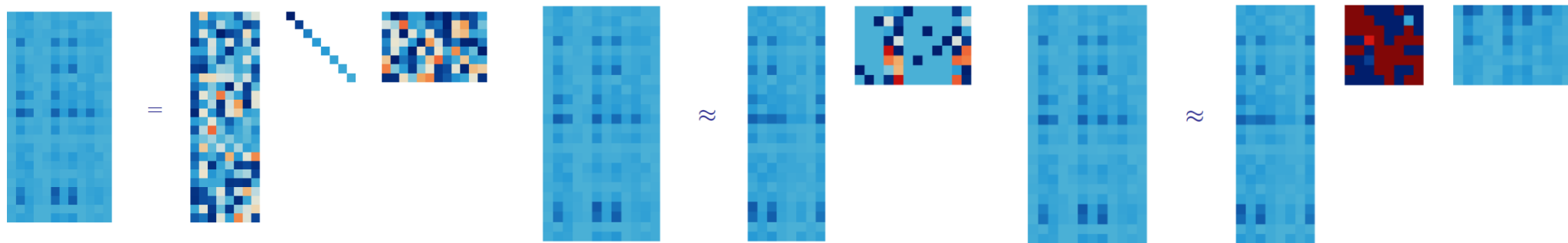
Sample SAR data from open Sandia set.

$$\text{im} = \text{magd\_sar} .* \exp(j * \text{phased\_sar})$$



Decay of data matrices (also common to other applications).

Rank-k svd gives best error bound, but ID, CUR useful for applications:  $A \approx CV^T$ , where  $C = A(:, J_c(1:k))$ ,  $V^T = [I_k \quad T_l] P^T$



Can build multiple representations (SVD, ID, CUR) efficiently with randomized schemes.



Some recent project directions..

(I) Machine Learning

Classification and regression problems. Parameter optimizations. Boosting and ensemble schemes. Data imputation and correction strategies. Anomaly detection.

(I) Antenna array systems

How to construct a 'fancy' signal in the far field using multiple transmitters in place of single large / expensive transmitter.

(II) PTSD detection

How to process audio data from medical interview, segment out the patient voice, extract relevant features, and assign likelihood score of emotion state or PTSD likelihood.

(III) Data compression for heterogeneous bundles, audio, and image sequences.

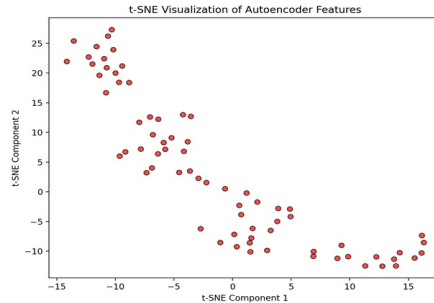
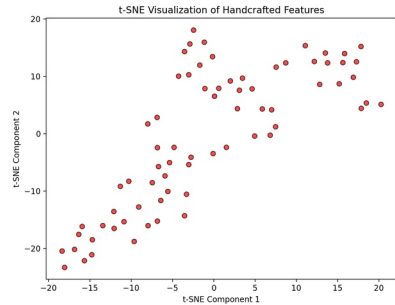
Heterogeneous data. Sets of similar signals (e.g. microphone arrays).

(IV) Gaussian process regression for multi-fidelity data applications.

(V) Accelerated implementation. (VI) Network data analysis. (VII) Multivariate time series.

# Machine learning

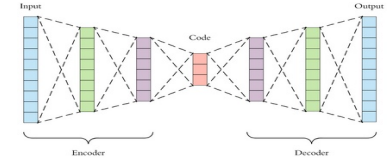
Implementations for several classification and regression problems with varying data sizes and quality (time series, text, audio, and imagery data applications).



Using autoencoders to supplement handcrafted feature sets by considering low dimensional data or feature set representation.

```
autoencoder = Model(inputs=input_layer, outputs=decoded)
encoder = Model(inputs=input_layer, outputs=encoded)
autoencoder.compile(optimizer='adam', loss='mean_squared_error')
history = autoencoder.fit(X_scaled, X_scaled,
                          epochs=250,
                          batch_size=16,
                          shuffle=True,
                          validation_split=0.2,
                          verbose=1)
```

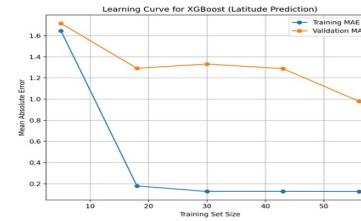
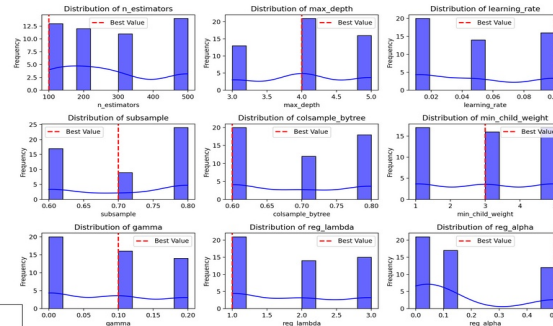
```
encoded_features = encoder.predict(X_scaled)
```



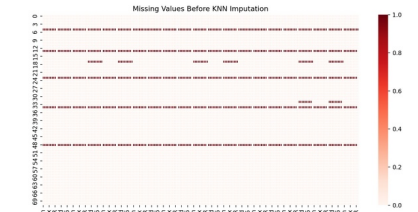
Feature set projections.

```
param_grid = {
    'n_estimators': [100, 200, 300, 500], # Number of boosting trees
    'max_depth': [3, 4, 5], # Depth of each tree (shallower trees prevent overfitting)
    'learning_rate': [0.01, 0.05, 0.1], # Controls how much each tree contributes
    'subsample': [0.6, 0.7, 0.8], # Use less data per tree to add randomness
    'colsample_bytree': [0.6, 0.7, 0.8], # Use fewer features per tree to prevent overfitting
    'min_child_weight': [1, 3, 5], # Minimum weight required to split a node
    'gamma': [0, 0.1, 0.2], # Penalizes small splits to avoid overfitting
    'reg_lambda': [1, 2, 3], # L2 regularization (reduces model complexity)
    'reg_alpha': [0, 0.1, 0.5] # L1 regularization (adds sparsity, feature selection)
}
```

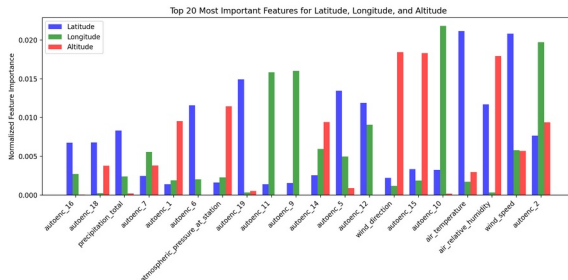
Parameter tuning via cross validation or Bayesian methods allows for optimized model performance.



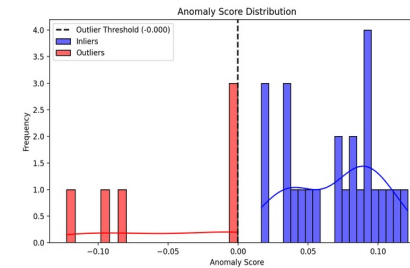
Learning curve for under and over fitting detection.



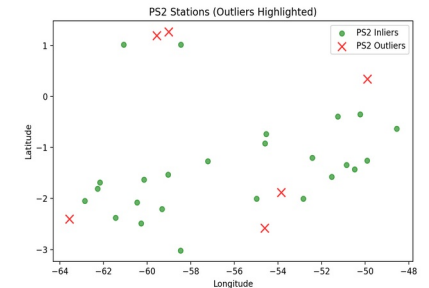
Missing data analysis and imputation.



Feature importance analysis, mapping to augmented feature matrix columns.

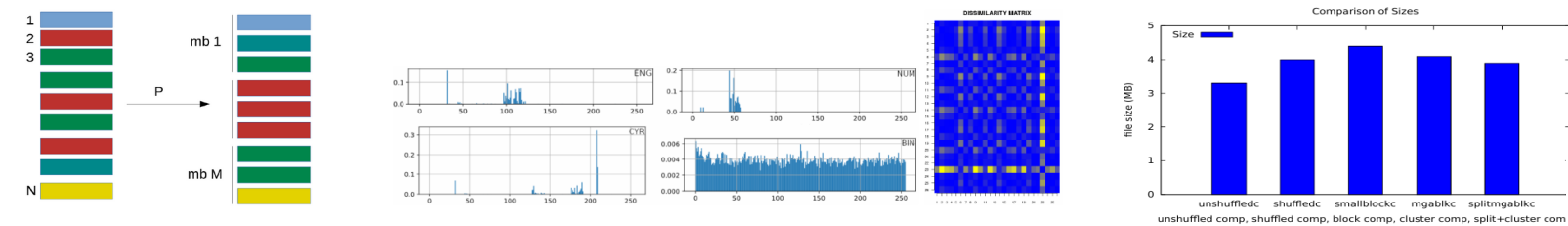


Anomaly identification and scoring.

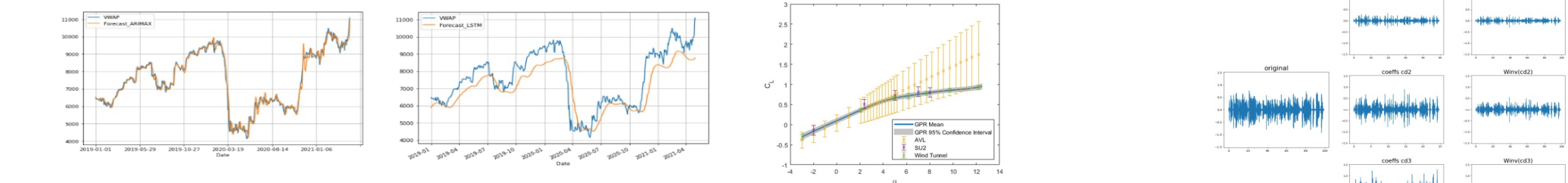


Cluster analysis.

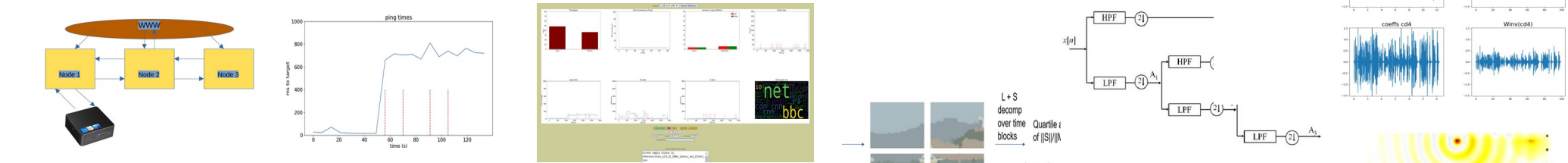
> Parallel data compression for heterogeneous data.  
 Achieving good compression ratios by clustering. Sets of similar signals (e.g. microphone arrays).



> Multivariate time series: prediction, data integration, anomaly detection.



> Network data analysis. Data collection and anomaly detection with FPGAs.

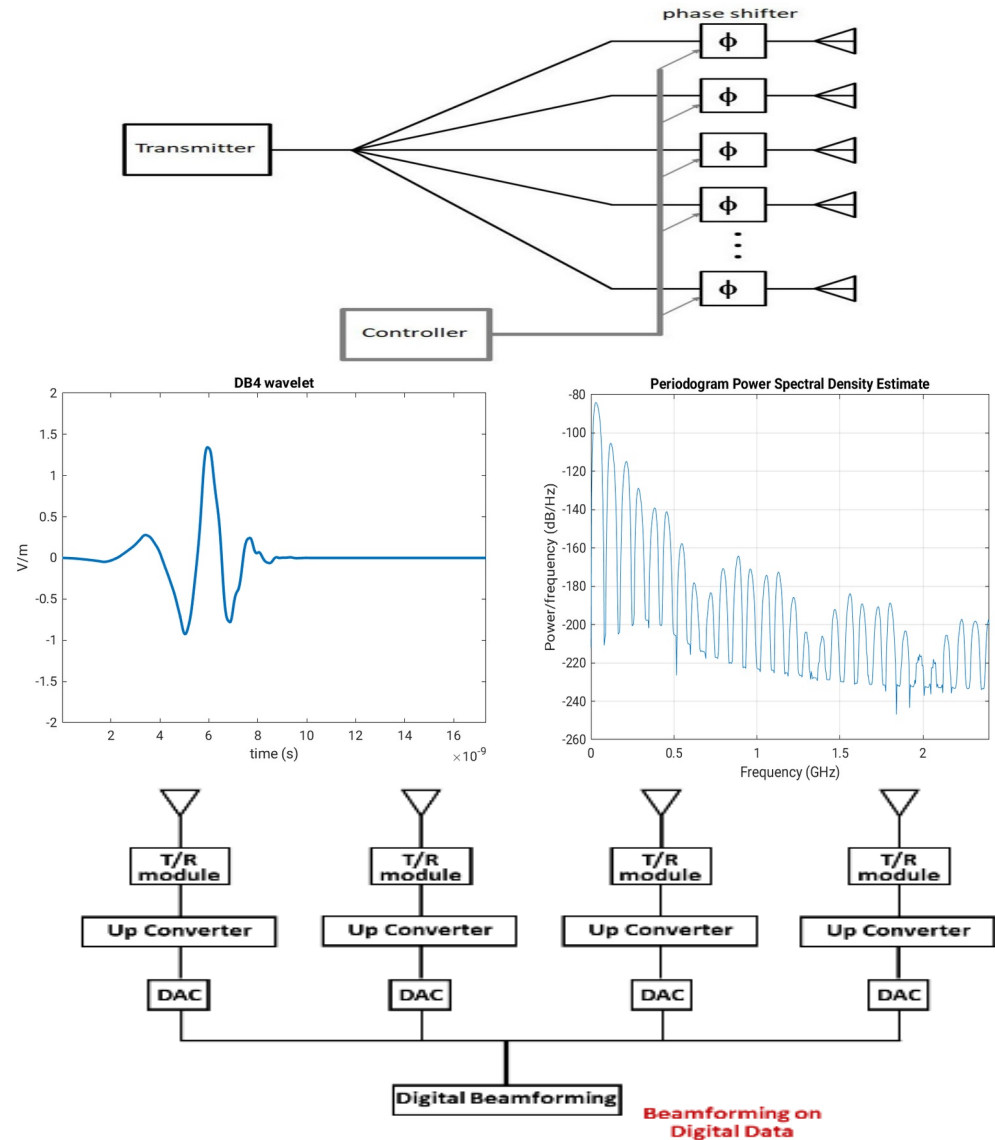
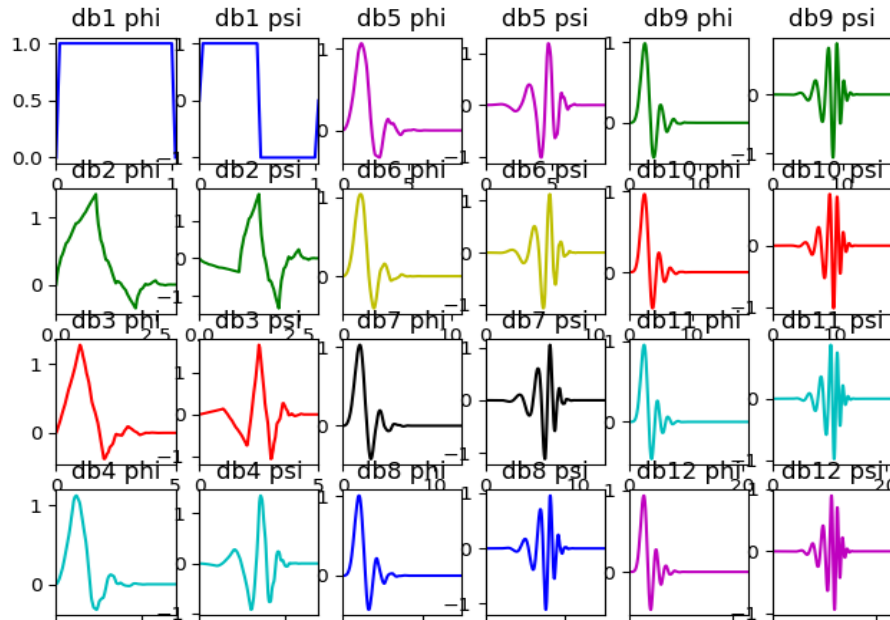


> Audio analysis / Antenna array systems / Video / etc.  
 How to process audio data from medical interview, segment out the patient voice, extract relevant features, and assign likelihood score of emotion state or medical indicators.  
 How to construct a signal in the far field using multiple transmitters?



# Antenna array systems

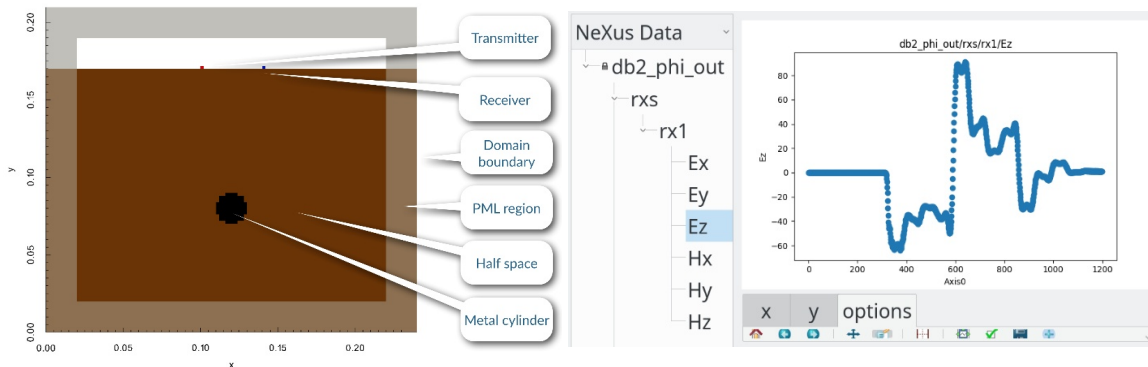
Project considered antennas which can emit compactly supported wavelets. (non-trivial hardware implementation).



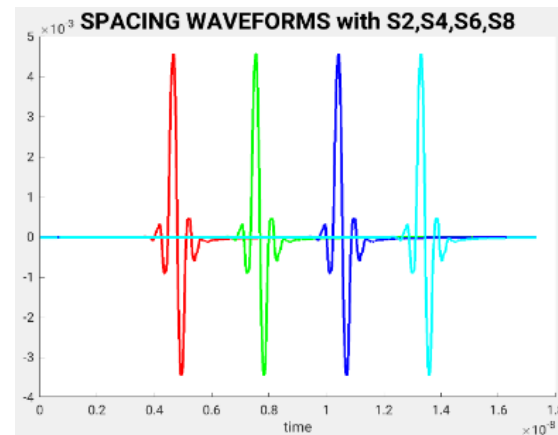
superimposed signal at receiver:

$$\sum_i [a_i x_i(b_i t - t_i) \star h_i] \star c_i$$

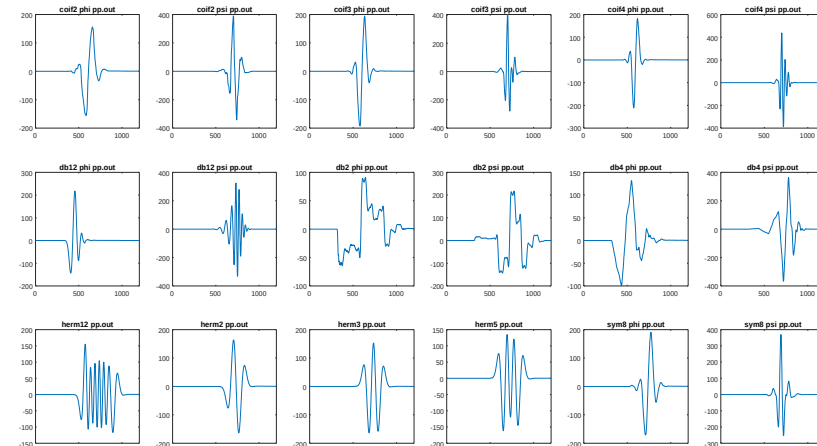
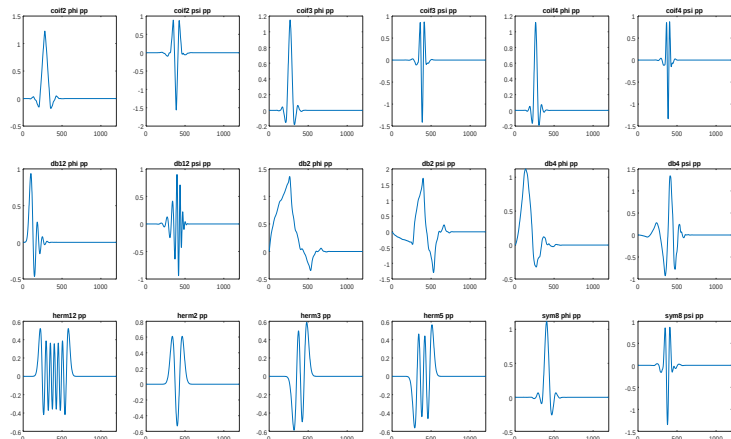
FDTD modeling for  
the environment.



Time delayed signals, convolved  
with antenna filter and propagation  
term (numerically determined).

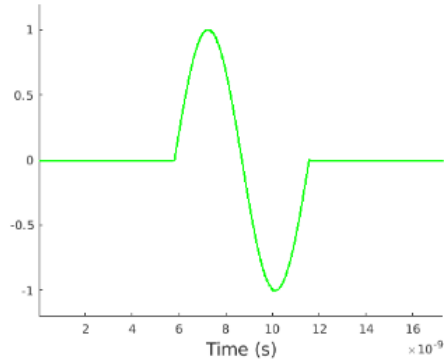


$$f_k(t) \approx \sum_{i=1}^n \alpha_i S^{[t_k + \delta_i]} w_i(t)$$

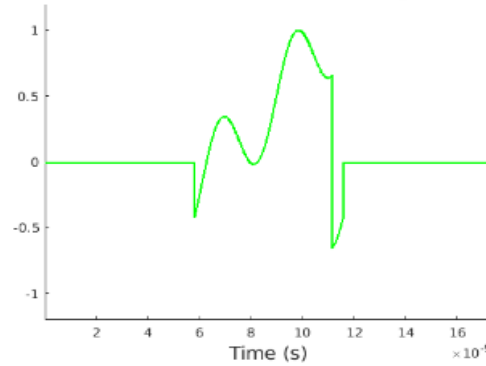




Portion of reference signal



Portion of reference signal



Desired reference signal at given window level.

$$\begin{bmatrix} \hat{w}_1(t_1, \delta_1) & \dots & \hat{w}_n(t_1, \delta_n) \\ \dots & \dots & \dots \\ \hat{w}_1(t_m, \delta_1) & \dots & \hat{w}_n(t_m, \delta_n) \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \dots \\ \alpha_n \end{bmatrix} \approx \begin{bmatrix} f_k(t_1) \\ \dots \\ f_k(t_m) \end{bmatrix}$$

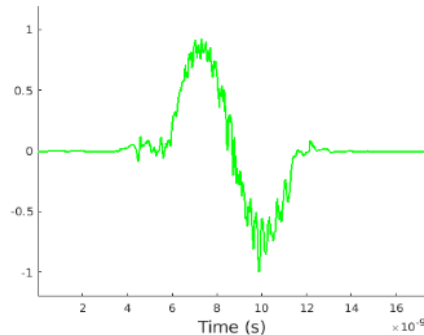
$$f_k(t) \approx \sum_{i=1}^n \alpha_i S^{\{t_k + \delta_i\}} w_i(t)$$

Use regularization and coordinate descent scheme to determine weighing factors and time delays.

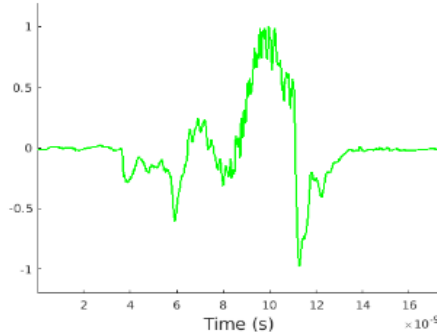
$S^{\{t_k + \delta_i\}} w_i(t)$  Time delayed signal via linear operator.

Can be solved as a regularized least squares problem + outer opt loop for remaining parameters.

L curve with IRLS l1-min



L curve with IRLS l1-min



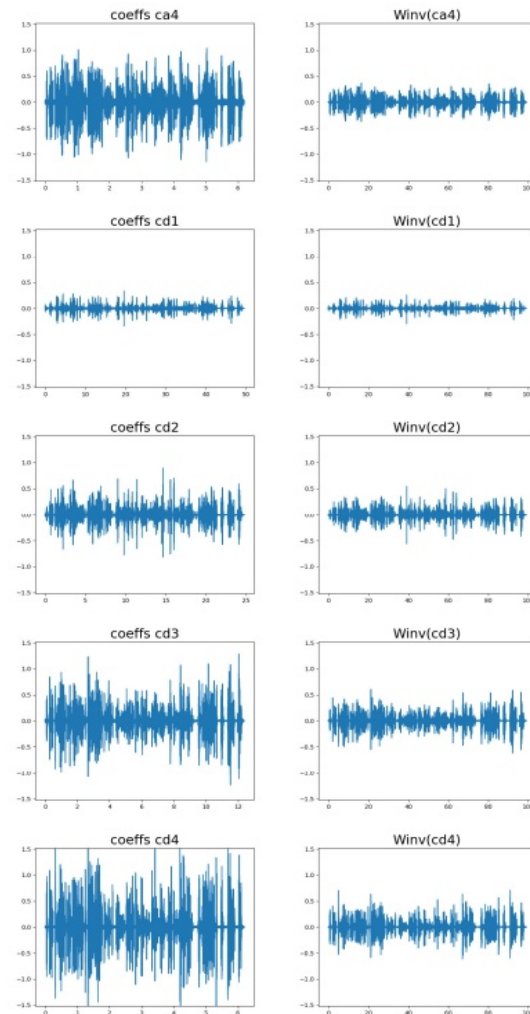
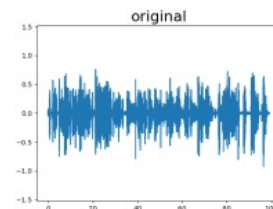
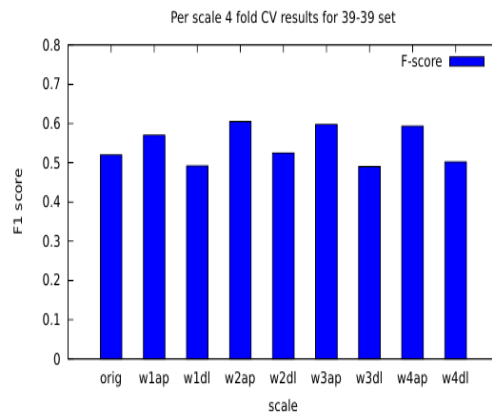
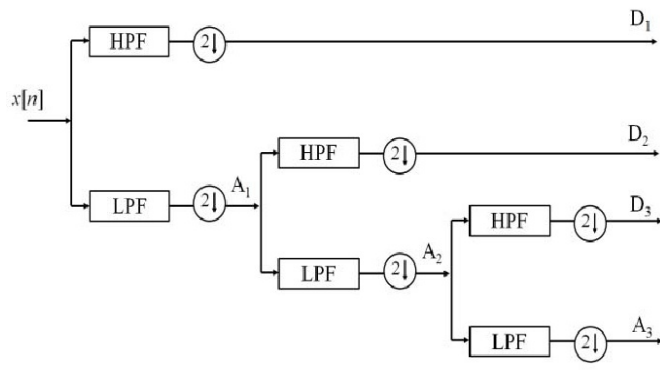
$$\min_x ||Ax - b||_l^l + \lambda \phi(x), \quad \phi(x) = ||x||_p^p$$

$(A^T S_k A + \lambda D_k^T D_k) x = A^T S_k b$  with iteration dependent diagonal matrices  $S_k, D_k$ .

# Application: emotion / PTSD detection from audio interviews.

Input: unsegmented audio interview, output: condition assessment score

Input audio can be decomposed into approximation and scaling coefficients using high / low pass filters and downsampling.



Different representations give different performance.

$x$  Original waveform

$a_{s_i} = W_i^{-1}[0; w_a]$  Coarse Approximation

$d_{s_i} = W_i^{-1}[w_d; 0]$  Fine Details

For original signal  $x$ , compute multi-level Wavelet transform:

- Collect approximation and detail coefficients.
- Apply inverse transform to obtain approximation and high level details.
- Repeat for 4 different bases (sharp and smooth).

We extract ~45 features per set, corresponding to spectral (e.g. MFCC), audio (e.g. tempo), and time series (e.g. mean auto-correlation) statistics.

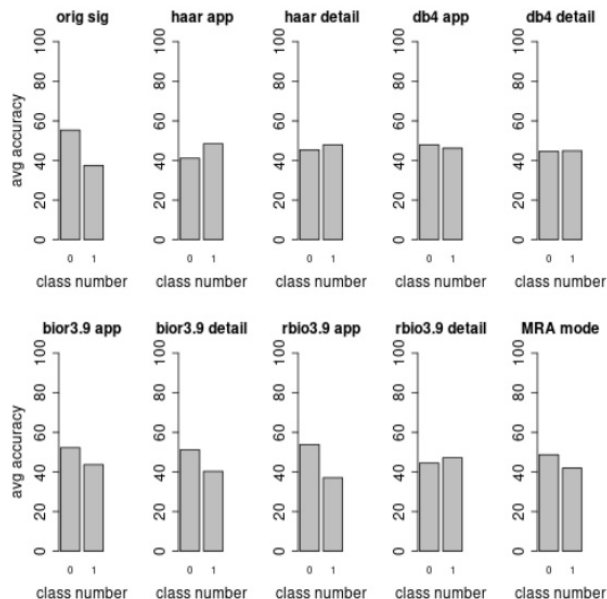
LibRosa, Aubio libraries

Algorithms (Java Weka, Scikit, TF) : Tree-based methods, LSTM (with all 9 feature sets in megatensor)

We obtain probabilities for class\_0 and class\_1 with each algorithm  $j=1,\dots,M$  and feature set  $k=1,\dots,9$  :

```
for features from file i in train_set:
    seq = [np.array(unt_rows[i]), np.array(wavlap_rows[i]),...,
            np.array(wav4dl_rows[i])]
    data.append(seq)

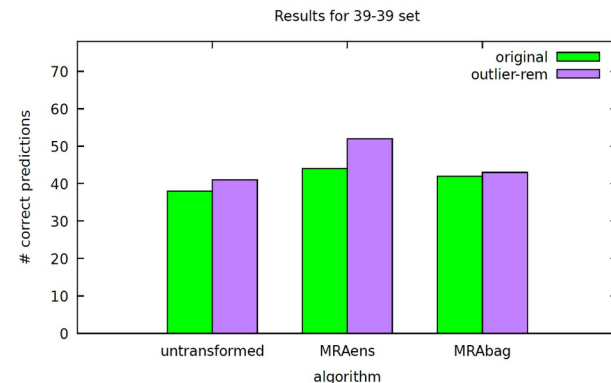
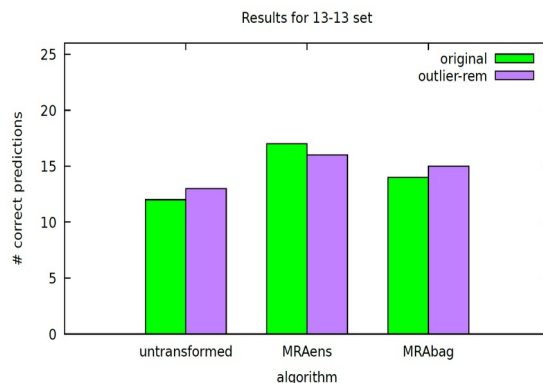
Xnew = np.zeros((num_files,nsets,nfeatures));
for i in range(0,num_files):
    for j in range(0,nsets):
        X_tr[i][j][:] = data[i][j];
```



Use an ensemble scheme, with weighted mean.

$$P(C_0 | A_j, S_k), P(C_1 | A_j, S_k)$$

$$\sum w_{\{j,k\}} P(C_l | A_j, S_k) / \sum \{w_{\{j,k\}}\}$$

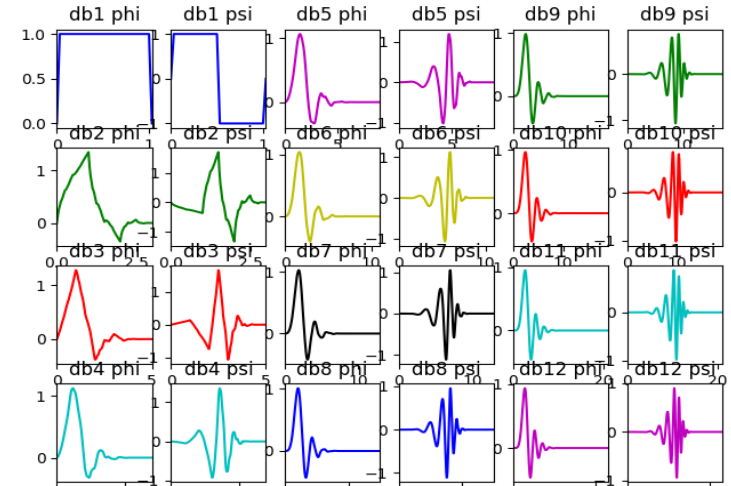
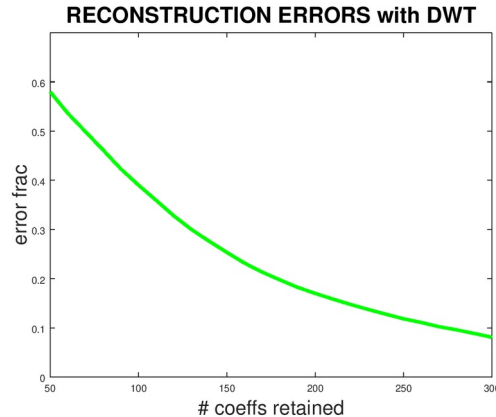
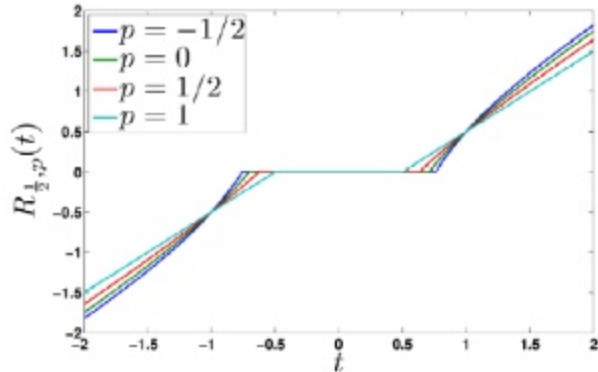


## Application: Data compression

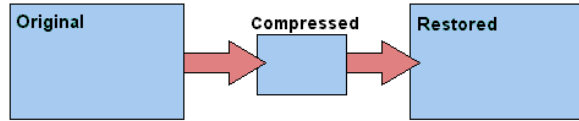
*Increasing adoption of high resolution content (e.g. 4K, high fidelity audio) various big data applications, effective parallel compression algorithms are becoming increasingly important. Both lossless and lossy compression are of interest (e.g. for text documents, where loss of information is not acceptable and for audio and image data where some losses are often plausible).*

Lossy compression based on transform / thresholding schemes for small coefficients. Can use e.g. CDF 9-7 wavelets and firm thresholding. Idea based on relation:

$$s \approx W_i^{\{-1\}}(Thr(W_i s))$$



Lossless compression based on reducing alphabet size and encoding frequently occurring symbols with fewer bits. Needed for e.g. text/numerical data, when loss of information is not acceptable.



Entropy of a set of elements  $e_1, \dots, e_n$  with probabilities  $p_1, \dots, p_n$  is:

$$H(p_1 \dots p_n) \equiv - \sum_{\forall i} p_i \log_2 p_i$$

Critically, one must seek to reduce data entropy to improve compression performance.

$$\text{Max when } p_1 = \dots = p_n = \frac{1}{n} \rightarrow H = \log_2(n)$$

Example 1:

A	$p_A=0.5$
B	$p_B=0.5$

$$\begin{aligned} H(A, B) &= -p_A \log_2 p_A - p_B \log_2 p_B = \\ &= -0.5 \log_2 0.5 - 0.5 \log_2 0.5 = 1 \end{aligned}$$

Example 2:

A	$p_A=0.8$
B	$p_B=0.2$

$$\begin{aligned} H(A, B) &= -p_A \log_2 p_A - p_B \log_2 p_B = \\ &= -0.8 \log_2 0.8 - 0.2 \log_2 0.2 \approx 0.7219 \end{aligned}$$

Burrows-Wheeler transform based compression.







BWT rearranges input to reveal patterns, MTF/RLE move common symbols to front, compress sequences of identical digits, EC (Huffman or Arithmetic coding), encodes remaining data in fewer bits.

BWT typically needs to be performed over small chunks due to expensive string sorting. Can use suffix arrays and take advantage of triangular structure .. towards  $O(n \log n)$ .

$SA[i] > 0 ? BWT[i] = T[SA[i]-1] : \$$

\$BANANA  
A\$BANAN  
ANASBAN  
ANANA\$B  
BANANA\$  
NASBANA  
NANASBA

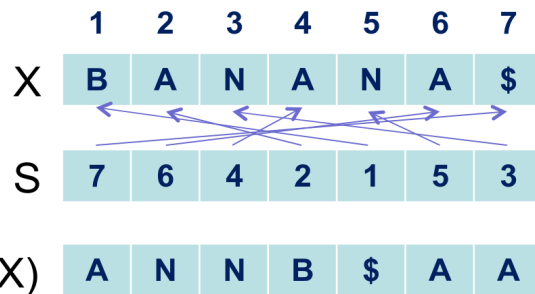
1 \$BANANA  
2 A\$BANAN  
3 ANASBAN  
4 ANANA\$B  
5 BANANA\$  
6 NASBANA  
7 NANASBA

1 banana\$  
2 anana\$  
3 nana\$  
4 ana\$  
5 na\$  
6 a\$  
7 \$

Sorting  
----->  
alphabetically

7 \$  
6 a\$  
4 ana\$  
2 anana\$  
1 banana\$  
5 na\$  
3 nana\$

BWT(X)



general purpose compressors (bzip2, lbzip2)

Need index permutation information from the sort. Developed  $O(n)$  counting sort with permutation information.

```

count = array of k+1 zeros
for x in input do
    count[key(x)] += 1

total = 0
for i in 0, 1, ... k do
    count[i], total = total, count[i] + total

output = array of the same length as input
for x in input do
    output[count[key(x)]] = x
    count[key(x)] += 1

return output
  
```

Move to front transform and Entropy coding: Huffman or Arithmetic coding.

Iteration	Index sequence	Symbol list
mississippi	12	abcdefghijklmnopqrstuvwxyz
mississippi	12,9	mabcdefghijklmnopqrstuvwxyz
mississippi	12,9,19	imabcdefghijklmnopqrstuvwxyz
mississippi	12,9,19,0	simabcdefghijklmnopqrstuvwxyz

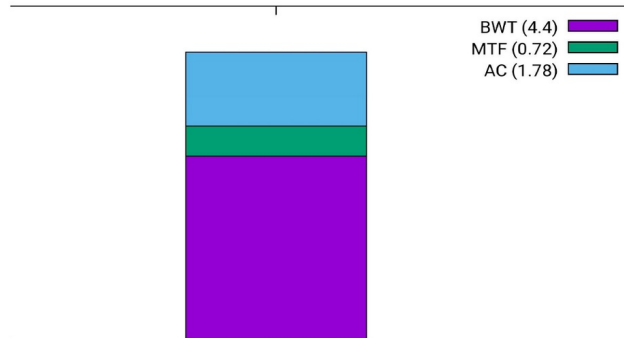
MTF is recency ranking scheme from symbol dictionary, converts to integer set.

Arithmetic coding represent input by a small interval (or some number within that interval). Better ratio than Huffman trees for heterogeneous inputs.

"c"	[0.0,	1.0	)
"o"	[0.0,	0.1	)
"m"	[0.03,	0.05	)
"p"	[0.034,	0.036	)
"r"	[0.0350,	0.0352	)
"e"	[0.03512,	0.03516	)
"s"	[0.035124,	0.035128	)
"s"	[0.0351272,	0.0351280	)
"o"	[0.03512764,	0.03512800	)
"r"	[0.035123748,	0.035127820	)
	[0.0351239912,	0.0351239056	)

Best done on larger chunks. Relatively slow. Need cumulative frequency count of encountered symbols; best done adaptively.

Timing breakdowns for Burrows-Wheeler compression (file 1)



encode\_symbol(symbol,cum\_freq)

range = high - low

high = low + range\*cum\_freq[symbol-1]

low = low + range\*cum\_freq[symbol]

decode\_symbol(encoded\_val,cum\_freq)

# find symbol such that the following is satisfied

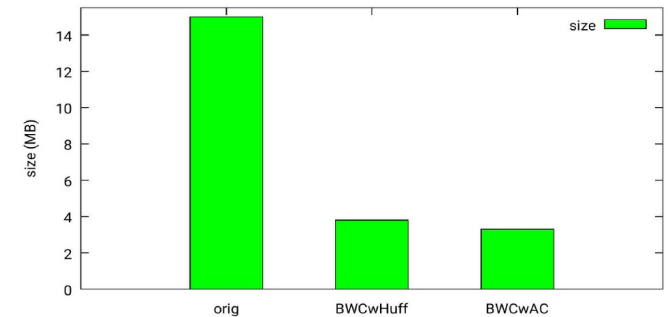
cum\_freq[symbol] <= (encoded\_val - low)/(high - low) < cum\_freq[symbol+1]

range = high - low

high = low + range\*cum\_freq[symbol-1] ; low = low + range\*cum\_freq[symbol]

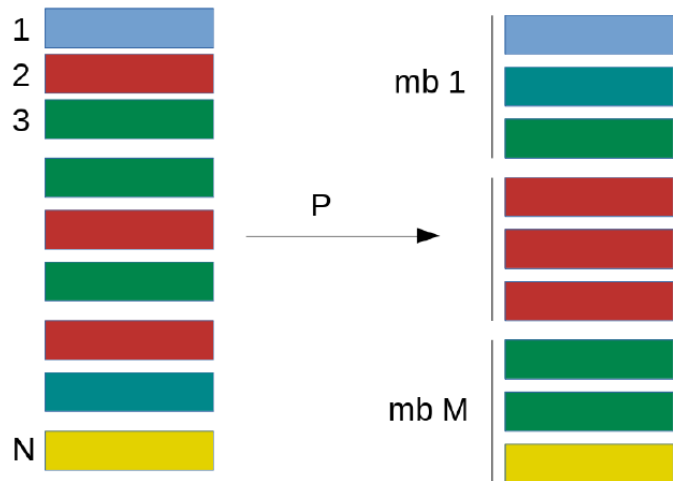
return x[symbol]

Sizes with Burrows-Wheeler compression (file 1)

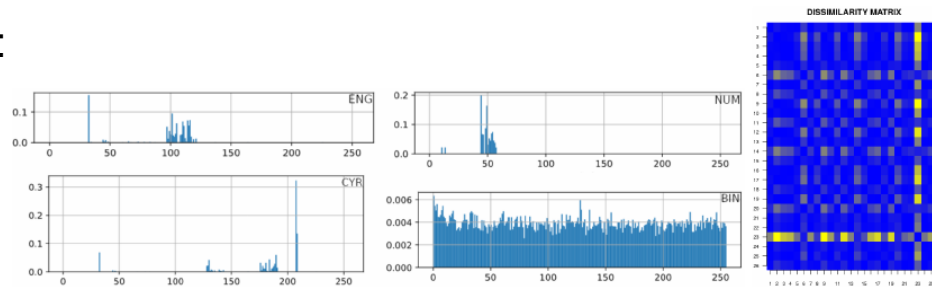


# Implemented enhancements for parallel compressor:

- 1) Subdivision into mega-blocks via symbol distribution clustering.

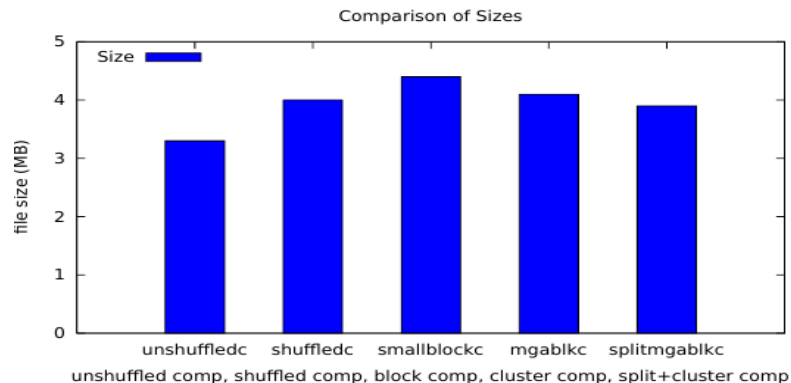


- 2)  $O(n)$  counting sort with indexing permutation output.



```
struct val_inds
{
    int val;
    int num_inds;
    int *inds;
};
```

before sort: 5 4 2 1 1 3 4 12 10  
after sort: 1 1 2 3 4 4 5 10 12  
inds after sort: 3 4 2 5 1 6 0 8 7



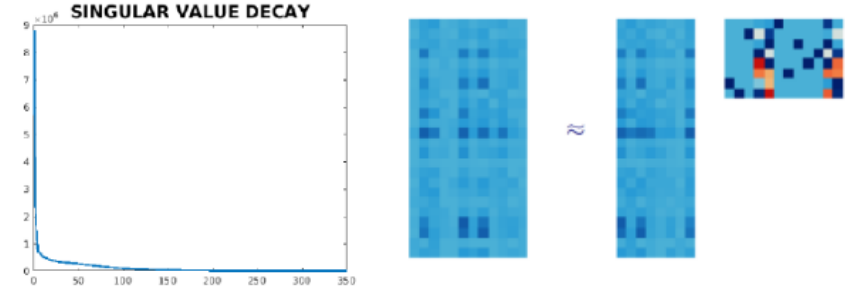
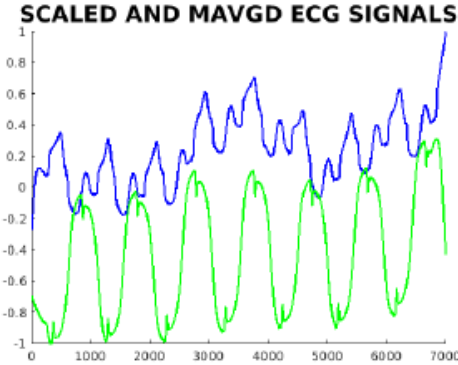
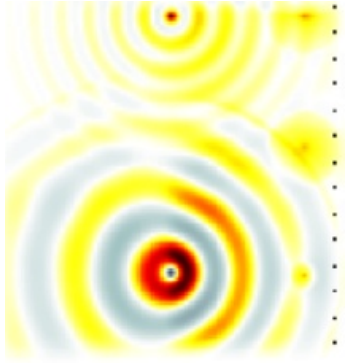
bananasale  
ananasale  
nanasale  
anasale  
nasale  
asale  
sale  
ale  
le  
e



ale  
ananasale  
anasale  
asale  
bananasale  
e  
le  
nanasale  
nasale  
sale

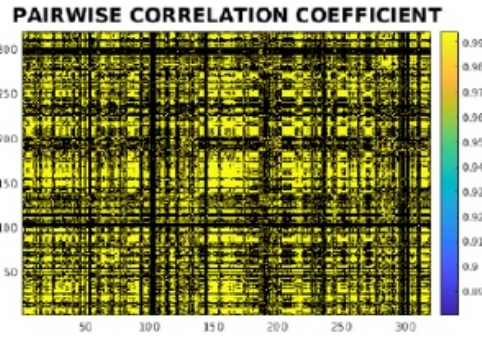
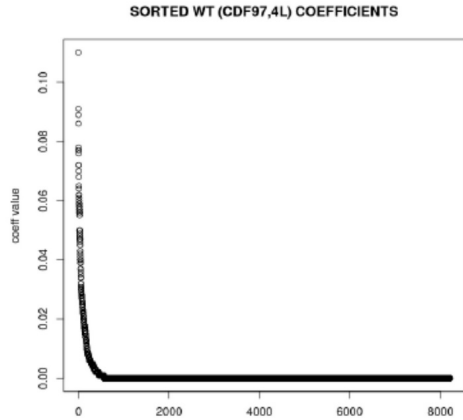
```
nbucket = 1, cur = (10)
nbucket = 2, cur = ananasale (97)
nbucket = 2, cur = anasale (97)
nbucket = 2, cur = asale (97)
nbucket = 2, cur = ale (97)
nbucket = 3, cur = bananasale (98)
nbucket = 4, cur = e (101)
nbucket = 5, cur = le (108)
nbucket = 6, cur = nanasale (110)
nbucket = 6, cur = nasale (110)
nbucket = 7, cur = sale (115)
```

# Application (similar signals): microphone array, ecg signals



The ID can be constructed from the partial pivoted QR factorization.

For remaining data, sorted abs values of transformed coefficients are exponentially decaying.



$$A(:, J_c) = \begin{bmatrix} Q_1 & Q_2 \end{bmatrix} \times_{r-k}^k \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = Q_1 S_1 + Q_2 S_2.$$

$$A(:, J_c) = Q_1 \begin{bmatrix} S_{11} & S_{12} \end{bmatrix} + Q_2 \begin{bmatrix} 0 & S_{22} \end{bmatrix} = \begin{bmatrix} Q_1 S_{11} & Q_1 S_{12} + Q_2 S_{22} \end{bmatrix}.$$

$$Q_1 S_1 = \begin{bmatrix} Q_1 S_{11} & Q_1 S_{12} \end{bmatrix} = Q_1 S_{11} [I_k \quad S_{11}^{-1} S_{12}] = C \begin{bmatrix} I_k & T_l \end{bmatrix},$$

$$A \approx C V^T, \quad \text{where } C = A(:, J_c(1:k)), \quad V^T = \begin{bmatrix} I_k & T_l \end{bmatrix} P^T$$

Applied on matrix transpose, yields a subset of the rows.

This allows only a portion of the most distinct channel data to be retained. Can then use high correlation modeling for remaining data.

```
sig_ref = log(E(ind,:));
p = polyfit(sig_ref, sig_new, 2);
yfit = p(1) * sig_ref.^2 + p(2) * sig_ref + p(3);
```

**Data:** Floating point data from multiple channels. Tolerance and pillar block parameters ( $\epsilon_{1,2,3}, L$ ), Wavelet transform, and thresholding function.

**Result:** Compressed representation of data for all channels.

Insert floating point data into matrix  $A$ , one channel per row.

Perform ID decomposition on the transpose of the matrix,  $A^T \approx A(J_r(1:k), :)V$  with rank chosen per  $\epsilon_1$  tolerance.

Set  $C = A(J_r(1:k), :)$  to be the subset of retained channels.

Form matrices  $\bar{M}, M_{num}, M_{sgn}$  from  $C$ .

**for**  $j = 1, \dots, k$  **do**

    Compute  $w_j = \text{transform}(C(j, :))$

$[v_j, I_j] = \text{sort}(\text{abs}(w_j), 'd')$

    Store permutation inds  $I_j$  from sort and signs of  $w_j(I_j)$  in  $M_{num}(j, :)$  and  $M_{sgn}(j, :)$ .

    Set  $\bar{M}(j, :) = \text{Thr}(v_j)$  per  $\epsilon_2$ .

**end**

Set  $M_E = 1e6, i = 0$ . Initialize  $E$  to hold subset (the pillars) of  $\bar{M}$  and  $F$  to hold linear fitting information.

**while**  $M_E > \epsilon_3$  **do**

    Add  $C(i+1, \dots, i+L, :)$  to  $E$ .

**for**  $j = i+L+1, \dots, k$  **do**

        Compute low order polynomial fit model between  $\log(\bar{M}(j, :))$  and each of the saved channels  $\log(E(i, :))$ .

        Record scaling factor  $s_j$ , modeling coefficients  $a, b$  and index to pillar model corresponding to smallest error against  $E(i, :)$  in  $F(j, :) = [a, b, si, i]$ .

        Record reconstruction error as  $e_j$ .

**end**

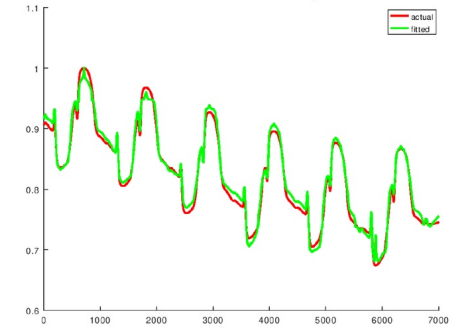
    Let  $M_E = \max(e_j), i = i+L$ .

**end**

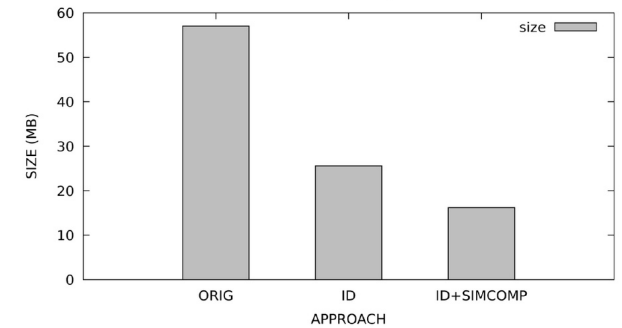
Lossless compress saved floating point data  $E$ , fitting coefficient set  $F$ , as well as the integer and bit sign matrices

$M_{num}$  and  $M_{sgn}$  and ID matrix  $V$ .

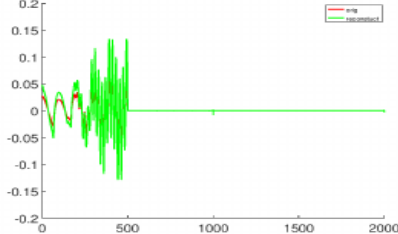
ACTUAL and FITTED signals



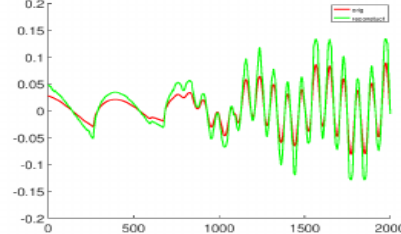
OUTPUT SIZES



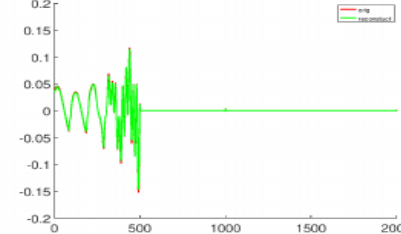
ORIG WAY COEFFS AND REC FROM PILLAR



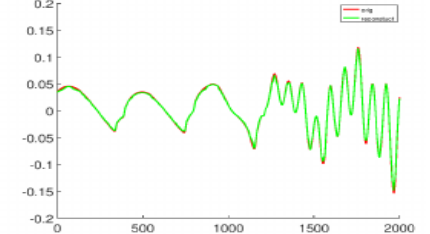
ORIG SIGNAL AND REC FROM PILLAR



ORIG WAY COEFFS AND REC FROM PILLAR

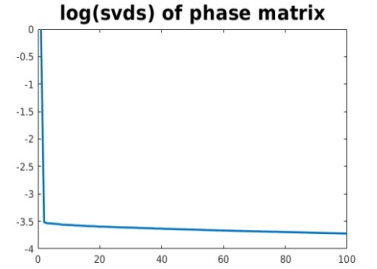
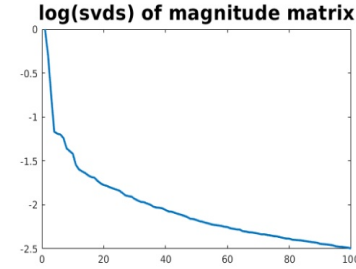
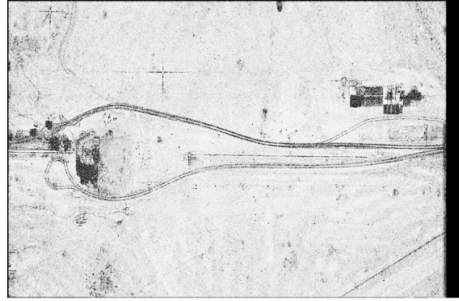
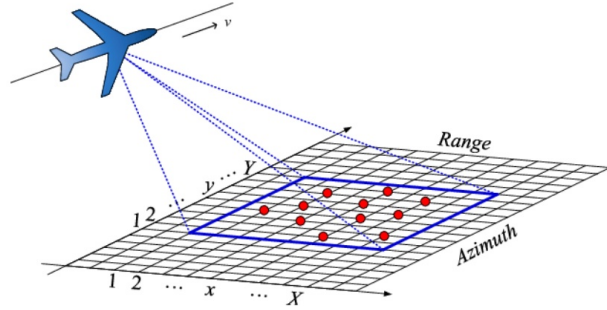


ORIG SIGNAL AND REC FROM PILLAR





# Analysis and compression of SAR data



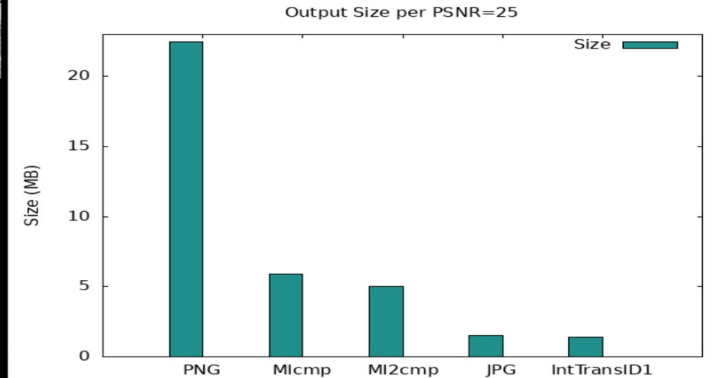
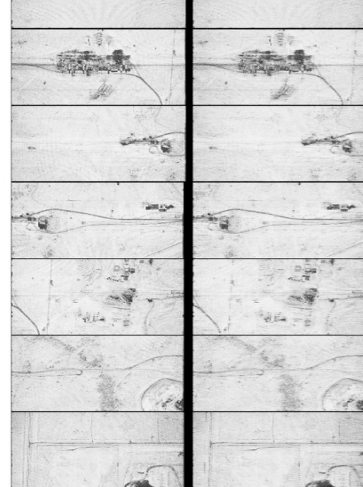
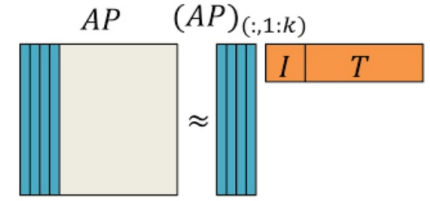
## Algorithm 1: SAR BLOCK PNG COMPRESS

**Input:** A set  $C = \{I_1, I_2, \dots, I_r\}$  of SAR images in PNG (or similar) format, block size  $l \times l$  and adaptive tolerance  $\epsilon$  or rank  $k$ .

**Output:** A compressed representation consisting of losslessly compressed ID components and scaling factors.

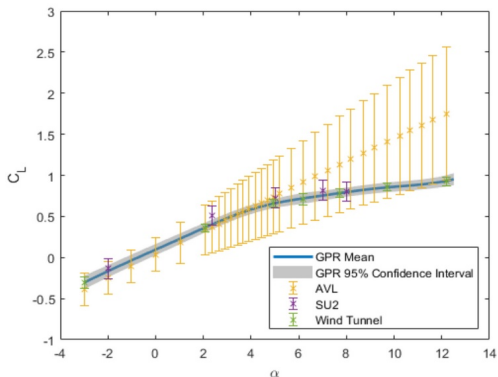
- 1 Break the image pixel set in  $l \times l$  blocks for a total of  $N_r$  blocks  $\{b_i\}$  representing the set.
- 2 Initialize transform matrix  
 $T_l = \text{round}(\text{dctmtx}[l] / \min(\min(\text{dctmtx}[l])))$ .
- 3 Apply transform and subtract smallest number from each block.
- 4 **for**  $i \leftarrow 1$  **to**  $N_r$  **do**
  - 5  $bt_i = T_l b_i$
  - 6  $mv_i = \min(\min(bt_i))$
  - 7  $bt_i = bt_i - mv_i$
  - 8  $M = [M; bt_i]$
- 9 Decompose matrix of transformed blocks  $M \approx M(:, I(1:k))Vt$  via pivoted QR factorization to tolerance level  $\epsilon$ .
- 10 Lossless compress remaining ID and scaling factors.

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	-1	-1	0	0	1	1	1	1
-1	0	0	1	1	0	0	-1	-1
-1	0	1	1	-1	-1	0	1	1
-1	1	1	-1	-1	1	1	1	-1
-1	1	0	-1	1	0	-1	1	1
0	1	-1	0	0	-1	1	0	0
0	1	-1	1	-1	1	-1	0	0



# Multi-fidelity with Gaussian Processes

Goal is to combine data with multiple fidelities (from simulations, testing) to build databases for aerodynamic modeling. Using Gaussian process regression (an interpolation method that pre-supposes a multi-normal Normal distribution on the target data).

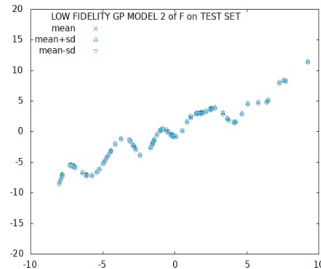
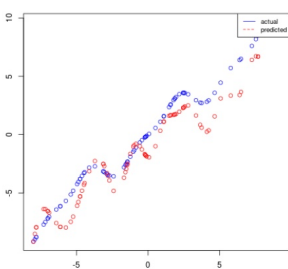
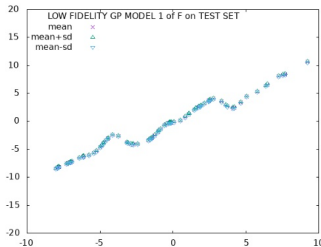
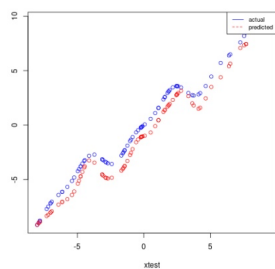


$$p(x|\mu, K) = 1/\sqrt{\det(2\pi K)} \exp \left[ -\frac{1}{2} (x - \mu)^T K^{-1} (x - \mu) \right]$$

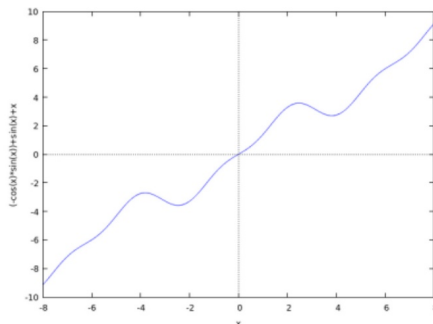
$$[y_{\{test\}}; y_{\{train\}}] \sim N([\mu_1; \mu_2], [\Sigma_{\{11\}}, \Sigma_{\{1,2\}}; \Sigma_{\{21\}} \Sigma_{\{22\}}])$$

$$y_{\{test\}} | y_{\{train\}} \sim N \left( \mu_1 + \Sigma_{\{12\}} \Sigma_{\{22\}}^{-1} (y_{\{train\}} - \mu_2), \Sigma_{\{11\}} \Sigma_{\{22\}}^{-1} \Sigma_{\{21\}} \right).$$

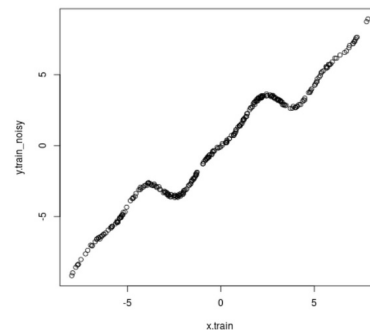
$$y_{\{test\}} | y_{\{train\}} \sim N \left[ \left( K_{\{s\}} (K + \lambda I)^{-1} y_{\{train\}}, K_{\{ss\}} - K_s (K + \lambda I)^{-1} K_s^T \right) \right]$$



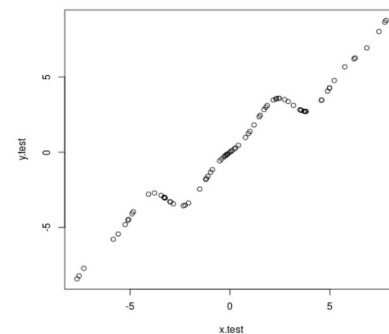
actual



train

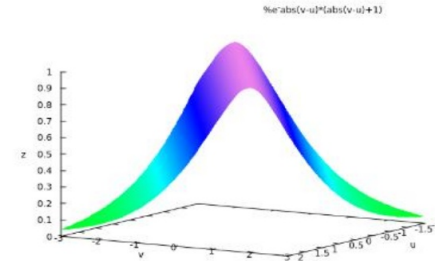
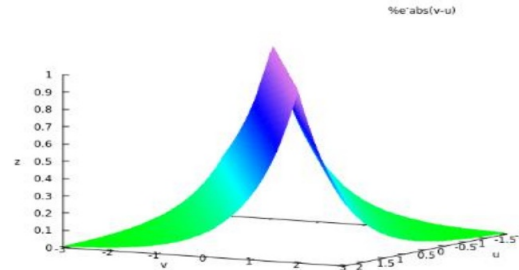
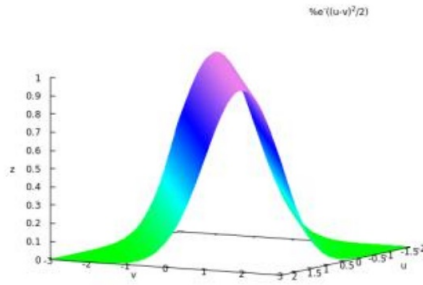


test

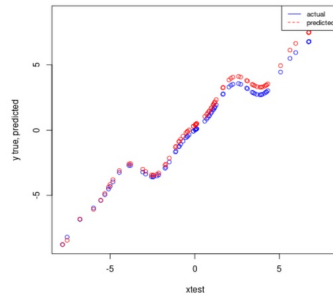
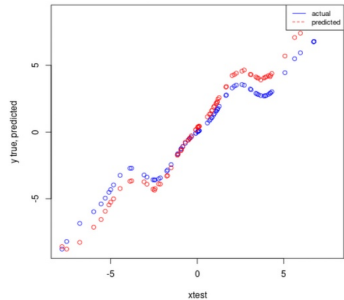


# Optimized Gaussian Processes

Different covariance 'kernels' with associated hyperparameters appropriate per different noise settings.



e.g.  $\text{Sigma}[i,j] = a \cdot \exp(-b \cdot (x1[i] - x2[j] - c)^d)$ ;  $\{a,b,c,d\}$  are the hyperparameters.



Optimization of kernel type and parameters often yields better prediction results.

```
for ( k in 1:ncol(kernels) ) {
# pick random num in 1-4 range
coord_to_opt = round(runif(1, 1, 4));

printf("==== iter = %d -> coord_to_opt = %d\n", k, coord_to_opt);
svals1 = c();
svals2 = c();
svals3 = c();
loglikevals = c();

for ( vind in 1:length(as) ) {
if (coord_to_opt == 1){
a = as[vind]; b = bsave; c = csave; d = dsave;
} else if (coord_to_opt == 2){
a = asave; b = bs[vind]; c = csave; d = dsave;
} else if (coord_to_opt == 3){
a = asave; b = bsave; c = cs[vind]; d = dsave;
} else if (coord_to_opt == 4){
a = asave; b = bsave; c = csave; d = ds[vind];
}

printf("test kernel %d\n", k);
gp = gp.solve( x.train_nt, y.noisy_nt, x.test_nt, kernels[[opt_kernel]], sigma2e, a, b, c, d )

# compute var sum
var_sums = colMeans(gp[['var']]);
sval1 = sum(var_sums);
print("sum of variances 1:");
sval1 = sum(var_sums);
print(sval1);
svals1 = c(svals1,sval1);

inds = which(x.test>5 & x.test<5)
var_sums2 = abs(var_sums[inds]);
print("sum of variances 2:");
sval2 = sum(var_sums2);
print(sval2);
svals2 = c(svals2,sval2);

diffs = gp[['mu']] - ytest_true;
diffs = abs(diffs[inds]);
sval3 = sum(diffs);
svals3 = c(svals3,sval3);

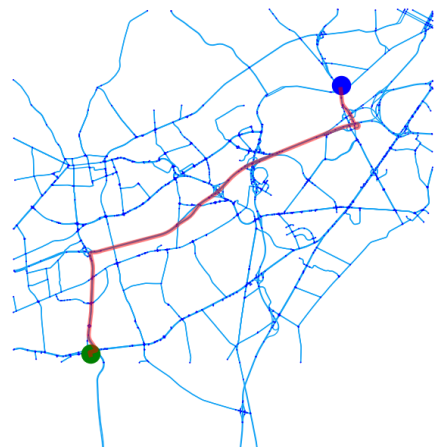
loglikeval = get_log_likelihood( x.train_nt, y.noisy_nt, x.test_nt, kernels[[opt_kernel]], sigma2e = 0, a, b, c, d );
loglikevals = c(loglikevals,loglikeval);
}
```

Developed optimized GP code performs several level of optimization to pick the optimal kernel and tune parameters for that kernel based on the train data. Basic approach based on randomized coordinate descent method.

```
## Function to evaluate Log-Likelihood
get_log_likelihood = function( x.train, y.train, x.pred, kernel, sigma2e = 0, a, b, c, d ) {
k.xx = kernel(x.train,x.train,a,b,c)
dims = dim(k.xx);
k.xx = k.xx + d*matrix(runif(dims[1]*dims[2]),dims[1],dims[2]);

Vinv = solve(k.xx + sigma2e * diag(1, ncol(k.xx)))
printf("size y.train:\n");
print(dim(y.train))
printf("size Vinv:\n");
print(dim(Vinv))
tyt = t(y.train);
printf("size tyt:\n");
print(dim(tyt))
val = -0.5*t(y.train)%*%Vinv%*(y.train - 0.5*log(norm(k.xx)) - length(y.train)/2*log(2*pi));
return (val);
}
```

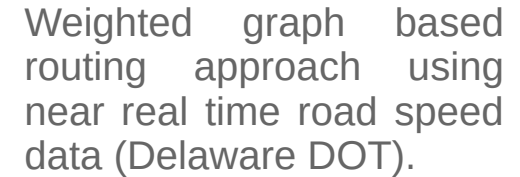
Evaluate model fit based on log likelihood and similar metrics.



```

etime data processing 0 of 375 processing 50 of 375 processing 100 of 375 processing 150 of 375 processing 200
1, lon2 = -75.743830,-75.587240 Converting multi-graph and extracting top path by min weight

```



ETT (hr): 0.43



# Accelerated implementation

Goal is to accelerate critical sub-parts of algorithmic implementation to enable application to bigger problem sizes and for faster parameter optimization.

```
// CUDA exp kernel
__global__ void kfunc_exp_kernel(double *x1, double *x2, double *Sigma,
                                const int M,
                                const int N,
                                const double a, const double b, const double c)
{
    int i = threadIdx.x + blockIdx.x * blockDim.x;
    int j = threadIdx.y + blockIdx.y * blockDim.y;
    if(i < M && j < N){
        Sigma[j*M+i] = a*exp(-b*pow(fabs(x1[i]-x2[j]),c));
    }
    return;
}
```

```
void get_idct(double **DCTMatrix, int M, int N){
    int i,j;
    #pragma omp for num_threads(4) collapse(2)
    for (i = 0; i < M; i++) {
        for (j = 0; j < N; j++) {
            if(i==0){
                DCTMatrix[j][i] = (double)(1.0/sqrt((double)N));
            } else{
                DCTMatrix[j][i] = (double)sqrt(2.0/(double)N)*cos((2*j+1)*i*M_PI/(2.0*N));
            }
        }
    }
}
```

```
// launch threads, one per byte array
pthread_t threadIds[4];
myargs = (struct arg *)malloc(4*sizeof(struct arg)); //array of structs, one per byte array

for(nb=0; nb<4; nb++){
    myargs[nb].byte_num = nb+1;
    myargs[nb].byte_arr = (unsigned char*)malloc(nints_in_file * sizeof(unsigned char));
    if(nb == 0){memcpy(myargs[nb].byte_arr, barr1, nints_in_file);} // or set addr
    if(nb == 1){memcpy(myargs[nb].byte_arr, barr2, nints_in_file);}
    if(nb == 2){memcpy(myargs[nb].byte_arr, barr3, nints_in_file);}
    if(nb == 3){memcpy(myargs[nb].byte_arr, barr4, nints_in_file);}
    ret = pthread_create(&threadIds[nb], NULL, processByteArray, (void *)&myargs[nb]);
    printf("after calling pthread create with id = %lu..\n", threadIds[nb]);
    if(ret != 0){
        printf( "Error creating thread %lu\n", threadIds[nb] );
    }
}
```

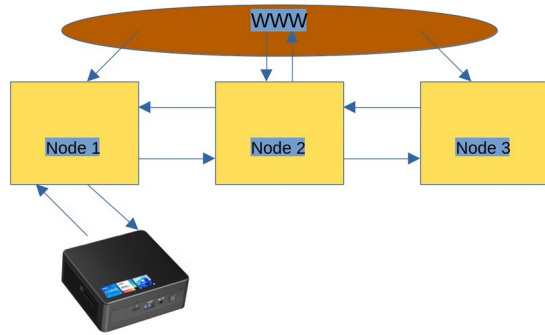
```
automodel = pm.auto_arima(tsdata,
                           start_p=1,
                           start_q=1,
                           test="adf",
                           seasonal=False,
                           trace=True)

ndays_ahead = 14;
tsdata_vals = tsdata.values;
tsdata_preds = automodel.predict(ndays_ahead);
tsdata_combined = np.concatenate((tsdata_vals,tsdata_preds));
date_list = list(df['Datetime'].map(lambda ss:ss.date()))+list((pd.timedelta_range(start='1
day',periods=ndays_ahead)+df.iloc[-1]['Datetime']).map(lambda ss:ss.date()));
```

Techniques with P-threads, OpenMP, CUDA, and time series based methods. Example: parallel BWT.

# Network data analysis / anomaly detection

A web or device-based service takes several rounds of network data which can be collected with a simple Linux based device, and text-based instructions, performs analysis on each uploaded data segment and creates outputs based on the user supplied instructions. This service can detect anomalies and changes in usage on a network.



Sample three node setup with network data collection and processing device.

```

suvoroni@mobl
Capturing on 'eth0'
30 t=6 of 7
** (tshark:990) 09:41:31.013994 [Main MESSAGE] -- Capture start
** (tshark:990) 09:41:31.014099 [Main MESSAGE] -- File: "data/
2 t=2 of 7
35
t=7 of 7
9 t=3 of 7
15 t=4 of 7
18 t=2 of 7
20 Running shark network sniffer... time = 09_41_36
cmd1 = tshark -i eth0 -s duration:7 -f pcap -w data/tshark_09_41_36.pcap
Running kernel based collection... time = 09_41_36
cmd2 = ./measure_nstats.pl eth0 7 10.55.222.212 09_41_36 data/k
my iface = eth0, time_pd_secs = 7, ping_ip = 10.55.222.212, cun
6.dat
Fields are:
t=1 of 7
Capturing on 'eth0'
22 ** (tshark:1184) 09:41:37.055725 [Main MESSAGE] -- Capture
** (tshark:1184) 09:41:37.055953 [Main MESSAGE] -- File: "data/
t=5 of 7
3 t=3 of 7
8 t=2 of 7
32
11 t=6 of 7
13 t=4 of 7
18 t=3 of 7
25 t=7 of 7

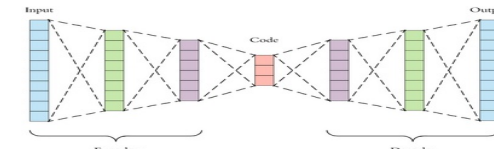
[ 5] local 10.55.222.212 port 20035 connected to 10.209.136.175 port 63164
[ ID] Interval      Transfer      Bitrate
[ 5] 0.00-1.00 sec    23.7 KBytes   194 kbits/sec
[ 5] 1.00-2.00 sec    64.0 KBytes   525 kbits/sec
[ 5] 2.00-2.37 sec    22.5 KBytes   581 kbits/sec
-----
[ ID] Interval      Transfer      Bitrate
[ 5] 0.00-2.37 sec    110 KBytes    381 kbits/sec
-----
Server listening on 20035 (test #2)
-----
[ 5] local 10.55.222.212 port 20035 connected to 10.209.136.175 port 64473
[ ID] Interval      Transfer      Bitrate      Jitter    Lost/Total Datagrams
[ 5] 0.00-1.00 sec    121 KBytes    990 kbits/sec  5.371 ms  311/413 (75%)
[ 5] 1.00-2.00 sec    115 KBytes    942 kbits/sec  11.693 ms  673/770 (87%)
[ 5] 2.00-3.00 sec    74.7 KBytes    612 kbits/sec  12.939 ms  627/690 (91%)
[ 5] 3.00-4.00 sec    69.9 KBytes    573 kbits/sec  15.857 ms  640/699 (92%)
[ 5] 4.00-5.00 sec    72.3 KBytes    592 kbits/sec  18.747 ms  630/691 (91%)
[ 5] 5.00-6.00 sec    61.6 KBytes    506 kbits/sec  16.292 ms  700/752 (93%)
[ 5] 6.00-6.94 sec    71.1 KBytes    623 kbits/sec  8.742 ms  414/474 (87%)
-----
[ ID] Interval      Transfer      Bitrate      Jitter    Lost/Total Datagrams
[ 5] 0.00-6.94 sec    586 KBytes    692 kbits/sec  8.742 ms  3995/4489 (89%) receiver
WARNING: Size of data read does not correspond to offered length
iperf3: error - unable to receive results: Bad file descriptor
Server listening on 20035 (test #3)

```

Server or mobile processing unit performs analysis on each data batch, following supplied instructions.

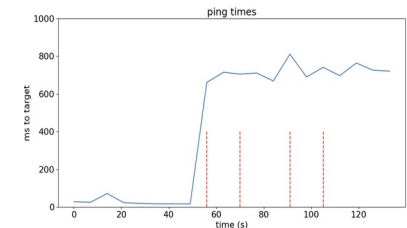
iperf_stats_10_01_57.txt	kernel_08_14_18.dat	ploss_10_05_48.dat	tshark_08_15_12.pcapng
iperf_stats_10_03_14.txt	kernel_08_14_24.dat	ploss_10_07_05.dat	tshark_08_15_18.pcapng
iperf_stats_10_04_31.txt	kernel_08_14_30.dat	ploss_10_08_22.dat	tshark_08_15_24.pcapng
iperf_stats_10_05_48.txt	kernel_08_14_36.dat	ploss_19_52_37.dat	tshark_08_15_35.pcapng
iperf_stats_10_07_05.txt	kernel_08_14_42.dat	ploss_19_54_20.dat	tshark_08_15_41.pcapng
iperf_stats_10_08_22.txt	kernel_08_14_48.dat	ploss_19_55_37.dat	tshark_08_15_47.pcapng
iperf_stats_19_52_33.txt	kernel_08_14_54.dat	ploss_19_59_55.dat	tshark_08_15_53.pcapng
iperf_stats_19_52_37.txt	kernel_08_15_00.dat	ploss_20_01_12.dat	tshark_08_15_59.pcapng

Heterogeneous data bundle from small analysis interval.

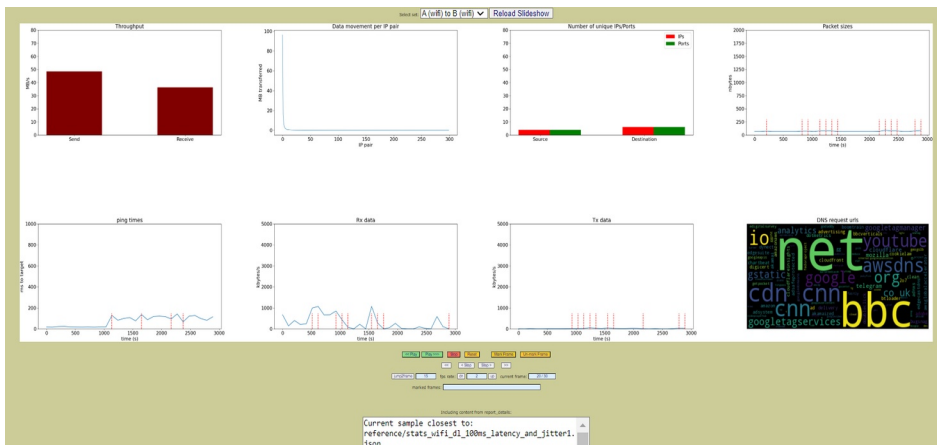


Feature mining for characterizing heterogeneous data bundles with statistics and autoencoder.

Based on the analysis, the processing node generates graphical output for each batch and for a collective multi-batch view, to allow domain experts to view results for outlier batches.



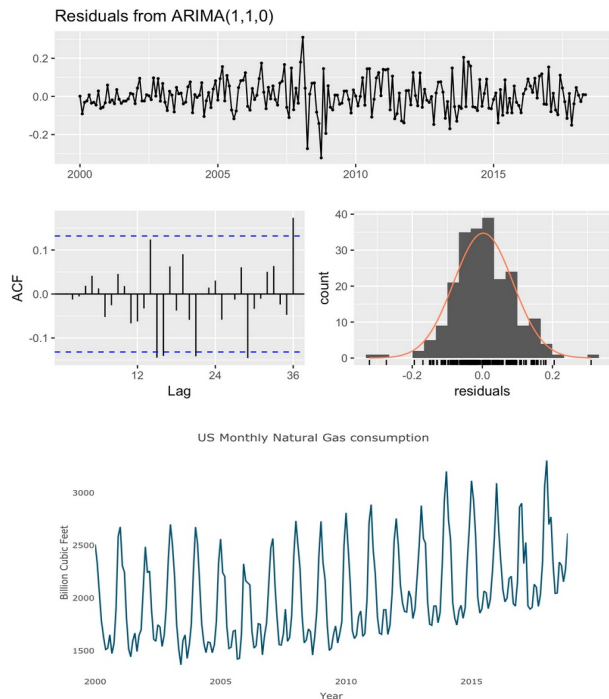
Change point / breakpoint detection in time series data (e.g. latency).





# Time series analysis / prediction

Many available methods for interpolation and prediction. Libraries in Python and R. Examples are ARIMA based codes and machine learning models (e.g. LSTM).



Time series with seasonal and trend patterns, can be handled with decompositions.

Autoregression, differencing, moving average.

$$AR(p) : Y_t = c + \sum_{i=1}^p \phi_i Y_{t-i} + \epsilon_t$$

Autoregression defines current value of series in terms of previous p lags.

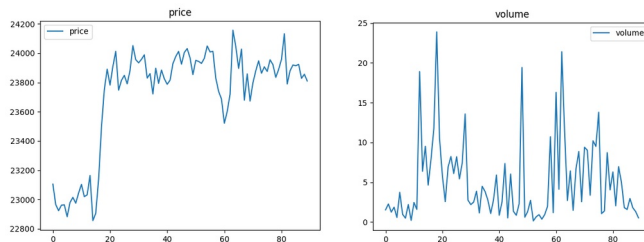
$$MA(q) : Y_t = \mu + \epsilon_t + \sum_{i=1}^q \theta_i \epsilon_{t-i}$$

Moving average captures patterns in residual terms.

$$ARMA(p, q) : Y_t = c + \sum_{i=1}^p \phi_i Y_{t-i} + \sum_{i=1}^q \theta_i \epsilon_{t-i} + \epsilon_t$$

Combination of AR and MA processes.

$$ARIMA(p, d, q) : Y_d = c + \sum_{i=1}^p \phi_i Y_{d-i} + \sum_{i=1}^q \theta_i \epsilon_{d-i} + \epsilon_t$$



```
X, y = create_sequences(scaled_data, seq_length, steps_ahead, feature_index)
print(f"Shapes after creating sequences - X: {X.shape}, y: {y.shape}")
```

```
split = int(0.8 * len(X))
X_train, X_val = X[:split], X[split:]
y_train, y_val = y[:split], y[split:]
```

```
input_layer = Input(shape=(seq_length, X.shape[2]))
lstm_units = hp.Int('lstm_units', min_value=32, max_value=128, step=16)
lstm_out = LSTM(lstm_units, return_sequences=True)(input_layer)
lstm_out = Dropout(hp.Float('dropout_rate', min_value=0.1, max_value=0.5, step=0.1))(lstm_out)
lstm_out = LSTM(lstm_units, return_sequences=True)(lstm_out)
lstm_out = Dropout(hp.Float('dropout_rate', min_value=0.1, max_value=0.5, step=0.1))(lstm_out)
```

```
# Attention mechanism
attention = Attention()([lstm_out, lstm_out])
attention_out = Concatenate()([lstm_out, attention])
```

```
# Flatten the output before dense layers
flat = Flatten()(attention_out)
```

```
# Ensure the dense layer output size matches steps_ahead
dense_units = hp.Int('dense_units', min_value=32, max_value=128, step=16) # Dynamically set output size
dense_out = Dense(dense_units, activation='relu')(flat)
```

```
# Reshape the output to match steps_ahead and features
output = Dense(steps_ahead)(dense_out)
output = Reshape((steps_ahead, 1))(output) # Predicting 1 feature (log_returns) for steps_ahead
```

```
model = Model(inputs=input_layer, outputs=output)
```

Many statistical choices for modeling univariate series.

ARIMA: A p-order AR process, d-degrees of differencing, and q-order MA process. No simple extension to multivariate case. Can use relatively simple VAR model instead:

$$Y_{1,t} = \alpha_1 + \beta_{11,1} Y_{1,t-1} + \beta_{12,1} Y_{2,t-1} + \epsilon_{1,t}$$

$$Y_{2,t} = \alpha_2 + \beta_{21,1} Y_{1,t-1} + \beta_{22,1} Y_{2,t-1} + \epsilon_{2,t}$$

$$Y_{1,t} = \alpha_1 + \beta_{11,1} Y_{1,t-1} + \beta_{12,1} Y_{2,t-1} + \beta_{11,2} Y_{1,t-2} + \beta_{12,2} Y_{2,t-2} + \epsilon_{1,t}$$

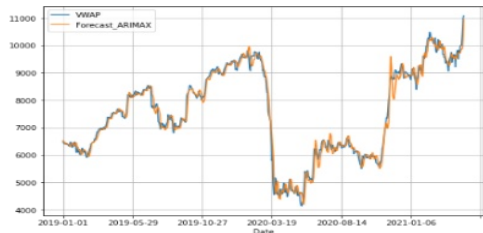
$$Y_{2,t} = \alpha_2 + \beta_{21,1} Y_{1,t-1} + \beta_{22,1} Y_{2,t-1} + \beta_{21,2} Y_{1,t-2} + \beta_{22,2} Y_{2,t-2} + \epsilon_{2,t}$$

```
model = VAR(price_and_vol)
model.information
model.neqs
model_fit = model.fit()
model_fit.coefs
pred = model_fit.forecast(model_fit.endog, steps=20)
```

Limited statistical methods for multivariate case drive interest towards machine learning approaches (e.g. LSTM), but there are challenges with optimal data formatting, model parameter optimization, and mechanism for multi-step ahead prediction.

# Multivariate time series predictions

Summary: For multivariate cases (where there are two or more series; for instance, price and volume data in finance, or medical pulse and oxygen saturation data), there are fewer available tools. Variance autoregressive models (VAR) are most common from statistics. There is a need for more advanced and efficient methods for different applications.



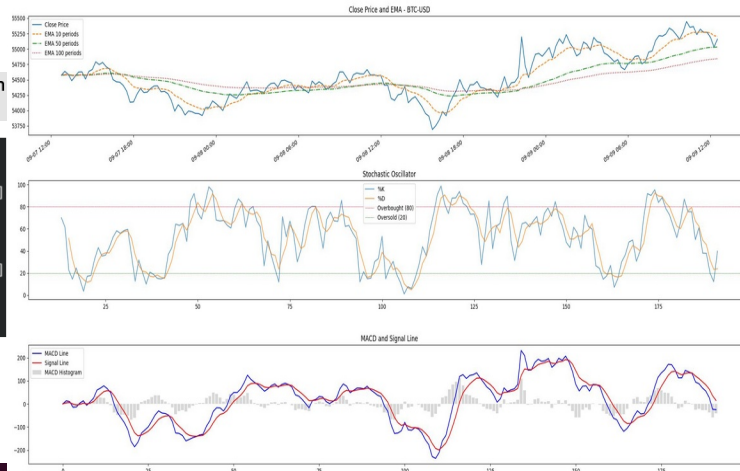
```
from sklearnex import patch_sklearn
patch_sklearn()
```

```
[[1.035e+02 1.300e+06]
 [1.028e+02 1.100e+06]
 [1.051e+02 1.400e+06]]

[[1.028e+02 1.100e+06]
 [1.051e+02 1.400e+06]
 [1.043e+02 1.500e+06]]

[[102.8 105.1 104.3]
 [105.1 104.3 106. ]]
```

Multi-dim data  
bundles



Statistical indicators and AI-based interpretation.

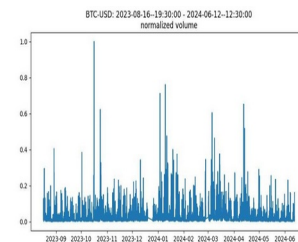
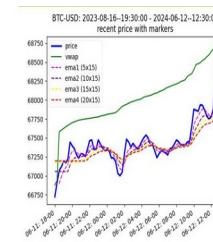
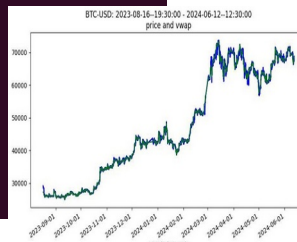
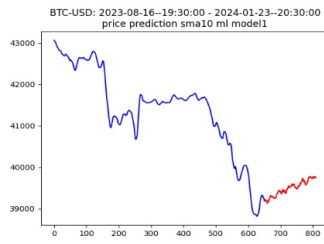
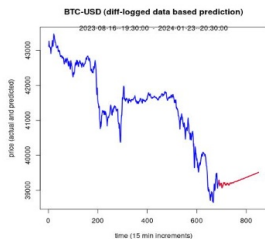
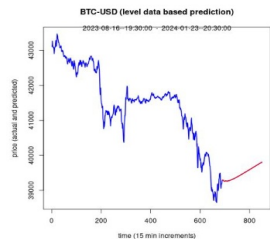
```
loglikeval = get_log_likelihood(x.train, y.train, x.test, kfunc_exp, sigma2e, a, b, c)
loglikevals = c(loglikevals, loglikeval);

}

if (coord_to_opt == 1){
    asave = as[which.max(loglikevals)]; b = bsave; c = csave; d = dsave;
} else if (coord_to_opt == 2){
    a = asave; bsave = bs[which.max(loglikevals)]; c = csave; d = dsave;
} else if (coord_to_opt == 3){
    a = asave; b = bsave; csave = cs[which.max(loglikevals)]; d = dsave;
}

}

end descent loop
```



```
input_layer = Input(shape=(seq_length, X.shape[2]))
lstm_units = hp.Int('lstm_units', min_value=32, max_value=128, step=16)
lstm_out = LSTM(lstm_units, return_sequences=True)(input_layer)
lstm_out = Dropout(hp.Float('dropout_rate', min_value=0.1, max_value=0.5, step=0.1))(lstm_out)
lstm_out = LSTM(lstm_units, return_sequences=True)(lstm_out)
lstm_out = Dropout(hp.Float('dropout_rate', min_value=0.1, max_value=0.5, step=0.1))(lstm_out)
```

```
# Attention mechanism
attention = Attention()([lstm_out, lstm_out])
attention_out = Concatenate()([lstm_out, attention])
```

LSTM with Bayesian parameter optimization.

## Select References

*Voronin, et. al., Compression approaches for the regularized solutions of linear systems from large-scale inverse problems, 2015.*

*Martinsson, Voronin. A randomized blocked algorithm for efficiently computing rank-revealing factorizations of matrices, 2016.*

*Thomison, et. al. A Model Reification approach to Fusing Information from Multifidelity Information Sources, 2017.*

*Stankovic, et. al. On a Gradient-Based Algorithm for Sparse Signal Reconstruction in the Signal/Measurements Domain, 2016.*

*Voronin, et. al., Multi-resolution classification techniques for PTSD detection, 2018.*

*Voronin, Mutlti-Channel similarity based compression, 2020.*

*Voronin et. al., Clustering and presorting for Burrows Wheeler based compression, 2021.*

*Voronin, SAR image compression with int-int transforms, dimension reduction, 2022.*