MATH 21-01 (Introductory Statistics, Voronin, S.), Exam #2 (100 points).

You may use a calculator and one page of notes. No cell phones or note taking devices. Please show your work for all problems. Clearly state your answer to each question. Sufficient written reasoning and calculation work is required to obtain full credit. Partial credit will be awarded where possible. Start: 9:30 AM. End: 10:20 AM. April 3rd, 2017.

Problem I, 20 pts

Suppose a procedure consists of tossing a fair coin three times. Let X equal the number of tails observed.

- (A, 4 pts) How many possible outcomes are there in this random experiment? List all of them.
- (B, 7 pts) Find the probability distribution of X. Define and sketch the CDF.
- (C, 4 pts) Find $P(X \geq 2)$ and $E[X]$.
- (D, 5 pts) Find $Var[X]$.

Problem II, 20 pts

For the following problems, write the answer in terms of factorials, combinations, and permutations as applicable, then simplify the calculation. Briefly explain your reasoning.

- (A, 5 pts) In how many ways can 9 books be split into two groups of 3 and 6 books respectively.
- (B, 3 + 5 pts) Out of 7 chemists and 5 poets, a committee consisting of 2 poets and 3 chemists needs to be formed. In how many ways can this be done if there are no restrictions? How does this change with the following combined restriction: two particular chemists cannot be on the committee and one particular poet must be on the committee?
- (C, 3 + 4 pts) Suppose 5 cards are pulled from a well shuffled 52 card deck. Find the probability of getting 4 aces and 1 jack. Find the probability that at least one ace is obtained.

Problem III, 20 pts

For the problems below, clearly define the events that you make use of and write down formulas used in your calculations with these events. Suppose you have two boxes of tea at home. Box I contains 6 black and 2 green high quality tea bags. Box II contains 25 black and 15 green tea bags. Before drinking tea, you take out two fair dice and roll them. If the sum of your rolls is a 3, 6, or 12, you proceed to choose one tea bag from box I. Otherwise, you choose one less good tea bag from box II. In all cases, the selection of one tea bag from the box is done randomly and the tea bags are well shuffled. The selected tea bag is then mixed with hot water and the tea drunk.

- (A, 10 pts) Find the probability that you wind up drinking green tea.
- (B, 10 pts) If you wound up drinking black tea, find the probability it came from box I.

Problem IV, 20 pts

Suppose X represents the number of minutes waiting for a bus at a stop, which is equally likely to come sometime in the next 75 minutes. Having lost your cell phone, you aim to estimate the wait time.

- (A, 1 + 4 pts) What is $E(X)$? Find $P(X < 15)$ and $P(X > 30)$.

Next, for a certain line of lottery tickets, let the probability of a lottery win of 100 dollars be 0.001 with any given ticket. Suppose each lottery ticket costs 2 dollars.

- (B, 5 pts) Define a suitable random variable and find the probability that it is necessary to spend more than 600 dollars on lottery tickets before winning the 100 dollar lottery. What's the expected number of dollars required to buy enough lottery tickets to win the 100 dollars?
- (C, 5 pts) Suppose you bought 50 tickets. Use the Binomial distribution to calculate the probability that less than two of the tickets will net you 100 dollar wins.
- (D, 5 pts) Suppose again, you bought 50 tickets. Use the Poisson distribution to estimate the probability that at least two of the tickets will net you 100 dollar wins.

Problem V, 20 pts

Assume that the heights of 3000 male students in a University are normally distributed with mean 68 inches and std deviation 3.0 inches. (This information is taken from the enrollment forms of all the students). Suppose now that 80 samples of 30 students each are obtained from the population, with replacement, and that the mean is computed for each sample.

- (A, 5 pts) What would be the mean and standard deviation of the resulting sample of means?
- (B, 15 pts) In how many samples would you expect to find the mean height to be between 66.6 and 68.5 inches? In how many samples would you expect to find the mean height to be less than 66.8 inches?

Some formulas

Baye's rule: $P(A_k|B) = \dfrac{P(A_k)P(B|A_k)}{\sum P(A_k)P(B|A_k)}$

Poisson probability: $P(X_p = k) = \dfrac{\lambda^k e^{-\lambda}}{k!}$

Binomial probability: $P(X_b = k) = \binom{n}{k} p^k (1-p)^{(n-k)}$