MATH 21-01 (Introductory Statistics, Voronin, S.), Final Exam (Spring 2017), 140 points.

You may use a calculator and two pages of notes (both sides). Clearly state your answer to each question. Please show your work for all problems, don't just write a numerical answer. Sufficient written reasoning and calculation work is required to obtain full credit. Partial credit will be awarded where possible.

Problem I, 20 pts

Suppose, the heights of 10 male and female University students are recorded in inches. They are reported as:
males: $(72, 72, 79, 74, 71, 61, 71, 68, 67, 72)$ and females: $(67, 81, 67, 62, 62, 64, 64, 69, 69, 70)$.

- (a, 8 pts) Find five number summaries for both sets. Plot a boxplots for each set. Are any values outliers?
- (b, 12 pts) Assuming the samples are independent, set up and carry out a statistical test at $\alpha = 0.05$ level to test the claim that males have greater average height than females at this University.

Problem II, 20 pts

A company claims that its machines fill containers of motor oil with a standard deviation of less than 2 milliliters (ml). A sample of 10 containers filled by the machine contained the following amounts (ml): $(999.01, 1000.78, 1001.02, 998.78, 1000.98, 999.25, 1000.56, 998.75, 1001.78, 999.76)$.

- (a, 10 pts) Find the 90 percent confidence interval for the population standard deviation. Describe how you would conduct a hypothesis test at $\alpha = 0.1$ to query the company's claim and the statistic you would use (you don't need to carry out the test).

In a poker game, five cards are drawn without replacement from a 52 card deck.

- (b, 4 pts) What's the probability that there are 4 aces amongst the 5 cards?
- (c, 6 pts) Find the probability that of the 5 cards drawn, 3 cards are of any one suit and 2 are of another different suit.

Problem III, 20 pts

Suppose a fair die is tossed. Let the events A and B represent getting an even number and getting a number less than or equal to 4, respectively.

- (a, 4 pts) Are A and B mutually exclusive? Find $P(A \cap B)$.
- (b, 6 pts) Are A and B independent? Prove your claim.
- (b, 10 pts) Find $P(A \cap B^c)$ and $P(A^c \cap B^c)$.

Problem IV, 20 pts

A medical test gives positive or negative outcomes when testing a patient for a certain disease. Suppose the chance of a false negative of the test is zero (showing negative when the subject has the disease). The chance of a false positive (showing positive when the subject does not have the disease) is 5%. The disease occurs only in a small percent of the population: two in 2000 get sick with the disease this year. Suppose a randomly selected person takes the test.

- (a, 15 pts) If the outcome of the test is positive, what's the probability that the patient has the disease? Clearly define any events you use and justify your answer.
- (b, 5 pts) How would the probability change if the test was improved to decrease the rate of false positives to 0.1%?

Problem V, 20 pts

The owner of a rare coin claims that the coin is biased. A coin is tossed 800 times. Let $H_0 : p = \frac{1}{2}$ with p the probability of heads.

- (a, 10 pts) How many heads must turn up in the 800 tosses to conclude that the coin is biased at the 5% confidence level?
- (b, 10 pts) Suppose 460 heads are obtained. If the alternate hypothesis is $H_1 : p > \frac{1}{2}$, estimate the p-value for the test using the normal distribution. Can you reject the null hypothesis at the $\alpha = 0.01$ level?

Problem VI, 20 pts

- (a, 10 pts) Suppose 4% of phone chargers of a certain brand sold at Walmart are defective. What is the probability that in a random sample of 30 chargers, at most one will be defective? How does your answer change if the Poisson distribution is used as an approximation?
- (b, 10 pts) Suppose multiple 52 card decks are mixed in a big bag. Cards are drawn from the bag, with replacement. What's the probability that more than 12 cards need to be drawn to get a jack?

Problem VII, 20 pts

Suppose that a city tests a manufacturer's pipe strength to see if it meets the requirements that the mean strength exceeds 2400 kg per linear meter. Suppose 50 pipes are tested and in this sample, a mean strength of 2460 kg per linear m is obtained with $s = 200$.

- (a, 10 pts) Estimate the probability of type II error if the true mean strength is 2450 kg per linear m. What is the power of the test in this case?

A manufacturer of flash drives has established that the median number of uses to failure for its drives is 5250 uses (plug in times, with read/write operation). A sample of 20 drives is obtained and tested until each fails. Suppose 13 of the 20 drives exceed 5250 uses before failure.

- (b, 10 pts) Is there evidence that the median failure usage exceeds 5250 uses at the 10% confidence level? (Hint: Use the large sample sign test).