1) A box of 20 components consists of 15 good, 5 faulty components. Suppose 3 components are picked at random.

(a) $P(\text{all 3 good}) = \dfrac{\#\text{ of ways to pick 3 good}}{\#\text{ of ways to pick 3}}$

$$= \frac{\binom{15}{3}}{\binom{20}{3}} = \frac{15!}{12!\,3!} \cdot \frac{3!\,17!}{20!}$$

$$= \frac{15!\,17!}{12!\,20!} = \frac{15\cdot 14\cdot 13\cdot 12!\cdot 17!}{12!\,20\cdot 19\cdot 18\cdot 17!}$$

$$= \frac{15\cdot 14\cdot 13}{20\cdot 19\cdot 18} \approx 0.399$$

Notice, using R we can evaluate this with the commands:

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choose(n=15,k=3)/choose(n=20,k=3).
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(b) $P(\text{all 3 faulty}) = P(0 \text{ out of } 3 \text{ good})$

$$= \frac{\binom{5}{3}}{\binom{20}{3}} \approx 0.0088$$

(c) $P(2 \text{ will be good}) = P(2 \text{ good}, 1 \text{ faulty})$

$$= \frac{\binom{15}{2}\binom{5}{1}}{\binom{20}{3}} \approx 0.46$$

(d) $P(\text{at least } 2 \text{ will be good}) =$

$$= P(2 \text{ good}) + P(3 \text{ good}) \quad \left(\begin{array}{c}\text{mutually}\\ \text{exclusive}\\ \text{events}\end{array}\right)$$

$$= \frac{\binom{15}{2}\binom{5}{1}}{\binom{20}{3}} + \frac{\binom{15}{3}\binom{5}{0}}{\binom{20}{3}}$$

$$\approx 0.46 + 0.40 = 0.86$$

Note in this case, direct evaluation is not harder than considering the complement event.

2) A coin is tossed 3 times.

X r.v., equal to # of heads in 3 tosses.

X can take values $0, 1, 2, 3$.

Sample space of outcomes. H = heads, T = tails:

$$\{\underset{1}{HHH}, \underset{2}{HHT}, \underset{3}{HTH}, \underset{4}{HTT}, \underset{5}{THH}, \underset{6}{THT}, \underset{7}{TTH}, \underset{8}{TTT}\}$$

note: # of outcomes $= 2^3 = 8$ (2 in each flip)

$P(X=0) = \frac{1}{8}$ (event TTT only)

$P(X=1) = \frac{3}{8}$ (HTT, THT, TTH)

$P(X=2) = \frac{3}{8}$ (HHT, HTH, THH)

$P(X=3) = \frac{1}{8}$ (HHH)

Thus, the probability distribution is:

x	0	1	2	3	sum
$f(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	1

$f(x) = P(X=x)$

pdf

probability density function

Next, we find expected value and variance of r.v. X:

$$E[X] = \sum_x x f(x) = \mu_X =$$

$$= 0 \cdot \frac{1}{8} + 1 \cdot \frac{3}{8} + 2 \cdot \frac{3}{8} + 3 \cdot \frac{1}{8} = 1.5$$

Note that the expected value is not a possible outcome since it's not a whole number. However, over many trials the sample mean tends to this value.

$$\sigma_X^2 = E[(x-\mu)^2] = \sum_x (x-\mu)^2 f(x)$$

$$= (0-1.5)^2 \frac{1}{8} + (1-1.5)^2 \frac{3}{8}$$

$$+ (2-1.5)^2 \frac{3}{8} + (3-1.5)^2 \frac{1}{8}$$

$$= 0.75 \quad \text{(variance)}$$

$$\sigma_X \approx 0.87 \quad \text{(std. deviation)}$$

3) Let X be binomial with parameters 4 and 0.5. What do the parameters represent? ($n=4$, number of trials; $p=0.5$, success probability in each trial).

$$E[X] = np = 4(0.5) = 2$$

$$\sigma_X = \sqrt{npq} = \sqrt{4(0.5)(0.5)} = 1$$

$$P(X = E[X]) = P(X=2)$$

$$= \binom{4}{2}(0.5)^2(0.5)^{4-2}$$

$$= \binom{4}{2}(0.5)^4 = \frac{4!}{2!2!}\,\frac{1}{16}$$

$$= \frac{4\cdot 3}{2\cdot 4\cdot 4} = \frac{3}{8}$$