(A) 6 sided die: 1, 2, 3, 4, 5, 6 $\Big\}$ 12 total pairs
coin flip: −1, 1

X r.v. records score (sum of toss and flip)

possible values: 0, 1, 2, 3, 4, 5, 6, 7

(B) PDF is $f(x) = P(X=x)$

x	combs	$P(x)$
0:	(1,−1)	1/12
1:	(2,−1)	1/12
2:	(1,1), (3,−1)	2/12 = 1/6
3:	(2,1), (4,−1)	2/12 = 1/6
4:	(3,1), (5,−1)	2/12 = 1/6
5:	(4,1), (6,−1)	2/12 = 1/6
6:	(5,1)	1/12
7:	(6,1)	1/12

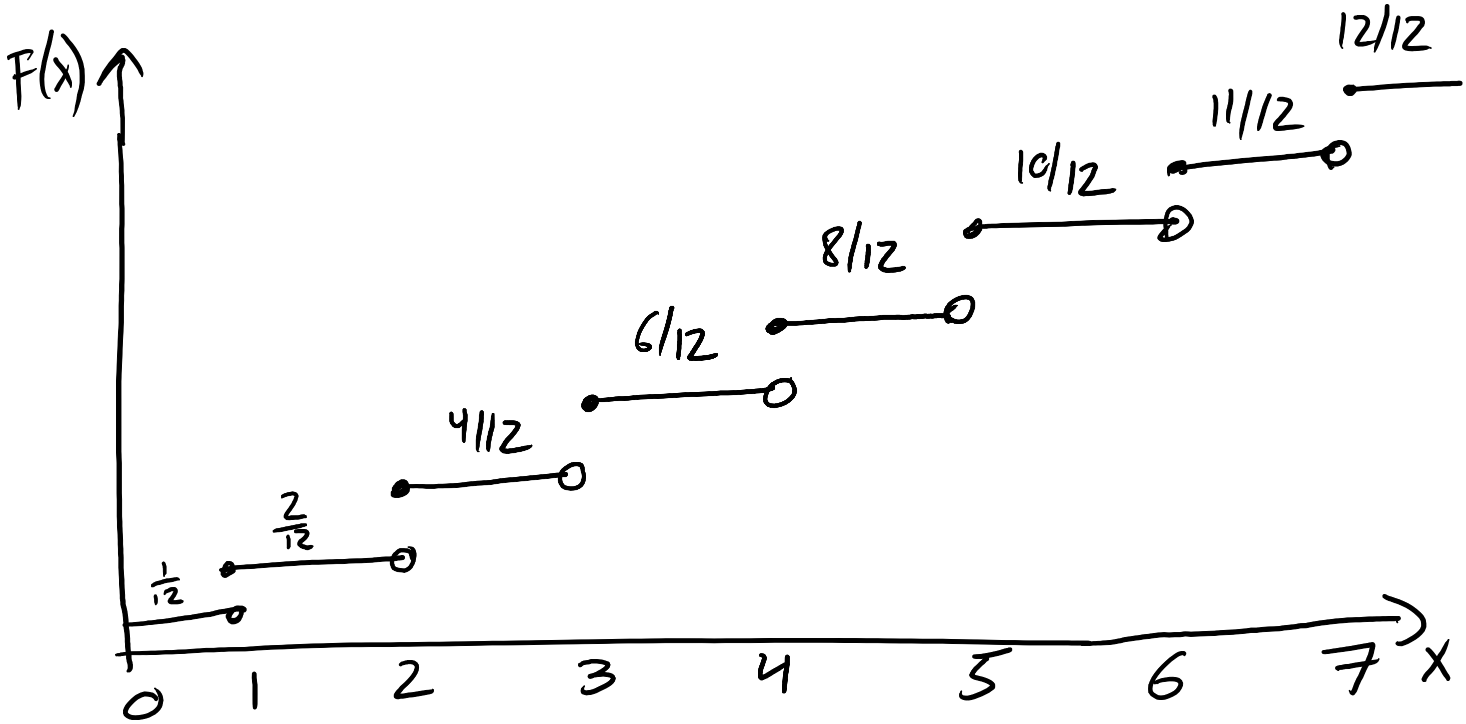
probability distribution of X

combination represents (dice value, coin value)

Note: $\sum_x f(x) = \sum_x P(X=x) = \frac{4}{12} + \frac{4}{6} = 1$

(C) CDF: cumulative distributive function

$F(x) = P(X \leq x)$



notice that $F(x) = 1$ for $x \geq 7$

also $F(x) = 0$ for $x < 0$.

If x is a whole #, $F(x) = \sum_{k=0}^{x} f(k)$, since X can only take integral values.

(D) $E[X] = \sum_x x f(x) = \sum_x x P(X=x)$

$$= 0 \cdot \frac{1}{12} + 1 \cdot \frac{1}{12} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6}$$

$$+ 6 \frac{1}{12} + 7 \cdot \frac{1}{12} = \frac{7}{2} = 3.5$$

$$\sigma_X^2 = E[(x-\overline{x})^2] = \sum (x-\overline{x})^2 f(x)$$

$$= (0-3.5)^2 \frac{1}{12} + (1-3.5)^2 \frac{1}{12} + (2-3.5)^2 \frac{1}{6}$$

$$+ (3-3.5)^2 \frac{1}{6} + (4-3.5)^2 \frac{1}{6} + (5-3.5)^2 \frac{1}{6}$$

$$+ (6-3.5)^2 \frac{1}{12} + (7-3.5)^2 \frac{1}{12}$$

$$\simeq 3.92$$

(E) The law of large numbers states

that $\overline{X}_n \xrightarrow{P} E[X]$ as $n \to \infty$

where $\overline{X}_n = \frac{1}{n} \sum_{i=1}^{n} x_i$

That is, if we play the game a large number of times and average the scores we get, the avg should go to the expected

value of X, where X is a r.v. following the same probability distribution as each X_i.
See R script for simulation.

(F) define success as getting a 3 on a roll of die. $n=10$ trials.

X is Binomial r.v. counting # of 3s in n rolls.

$p =$ success prob. $= \frac{1}{6}$; $q = 1-p = \frac{5}{6}$

$$P(X=4) = \binom{10}{4} p^4 q^6 = \frac{10!}{4!6!}\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^6$$

$$\approx 0.054$$

(G) success defined as resistor deviating by more than 2%. $p = 0.003$.

X binomial r.v. counting # resistors deviating more than 2% in n trials.

$n=500$

$$P(X=0) = \binom{500}{0} p^0 q^{500}$$

$$= (.997)^{500} \approx 0.22$$ (quite small probability)

(H) How to get sum total of 7 in two rolls: (1,6), (6,1), (3,4), (4,3), (5,2), (2,5) success: get sum 7

$$P(\text{sum 7 on two rolls}) = \frac{6}{36} = \frac{1}{6} = p$$

$n=5$ (5 rolls of two dice)

X binomial r.v. recording # of sum 7s in n trials

$$P(X \geq 1) = 1 - P(X<1) = 1 - P(X=0)$$

since X can take only values $0, 1, \dots, n$.

$$P(X=0) = \binom{5}{0} \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^5 \approx 0.40$$

$$\Rightarrow P(X \geq 1) \approx 1 - 0.40 = 0.60$$

At each trial, two dice rolled.

(I) $n=10$ (10 questions)

success: answer question correctly

$p=\frac{1}{4}$ (one out of 4 answers correct)

X binomial r.v. recording # correct answered in n trials (in n questions).

$$P(X \le 3) = P(X=0)+P(X=1)+P(X=2)+P(X=3)$$

$$P(X=k) = \binom{10}{k}\left(\frac{1}{4}\right)^k\left(\frac{3}{4}\right)^{10-k}$$

We can evaluate the sum in R using the pbinom command for cumulative distribution function.

$\Rightarrow$ pbinom(3, size=10, prob=$\frac{1}{4}$) $\simeq .776$