MATH 21-01 (Introductory Statistics), HW6 (100 points). Due: 03/15/2017 in class.

- (1) Evaluate the following summations:

$$\sum_{i=10}^{100} \left[\frac{1}{4}\right]^i \quad \text{and} \quad \sum_{i=4}^{\infty} \left[\frac{1}{2}\right]^i$$

- (2) State the definition of a Geometric random variable. What are the possible values it can take? Write down the PDF and derive the CDF. Suppose $X \sim \text{Geo}(p)$ with $p = 0.1$. Find $P(X \geq 12)$. As an example of this situation, consider the probability p of winning a small lottery. Then $P(X \geq 12)$ would be the probability that at least 12 tickets need to be bought for a win to occur.

- (3) Derive Bayes' theorem:

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

  Explain in words its significance.

- (4) Suppose one has 3 tanks of fish, containing blue and gold fish (only two kinds of fish in each tank). Each tank has more than 10 fish. Tank 1 has twice as many blue fish as gold fish. Tank 2 has 3 gold and 10 blue. Tank 3 has four times as many gold fish as blue fish. Suppose we choose a tank at random (equal probability of choosing any tank). From this tank we select 5 fish, one at a time, replacing each fish into the tank after it has been selected. The result is that we have 3 gold and 2 blue fish. Find the probability that given this distribution of the 5 fish, we were using tank T (where $T = 1, 2, 3$). Hint: make use of Bayes' rule and the Binomial distribution.

- (5) In a small county of Arkansas, 10 percent of registered voters belong to party I, 70 percent to party II, and 20 percent to party III. On a given new law proposal, 10 percent of party I members support it, 85 percent of party II members support it, 50 percent of party III members support it. The rest in each party do not support the law. Suppose a randomly chosen registered voter is sampled in this county. Find the probability that she or he supports the new law.

- (6) Let $X \sim \text{Geo}(p)$. Prove that for any integers $j, k \geq 1$, $P[X > (j + k) \mid X > j] = P[X > k]$. Interpret in words this result.

- (7) Suppose that we have assembled a group of 8 women, 6 men, 4 boys, and 5 girls. From this group, we want to select 2 men, 4 women, 3 boys, and 3 girls. In how many ways can we do this if (a) there is no restriction in our selection and (b) if a particular woman and a particular man must be selected. Hint: order in selection is not relevant.

- (8) A book shelf has 6 distinct math books and 4 distinct history books. The books are randomly re-arranged. Find the probability that 3 particular math books will be together (no other books between them) on the shelf. Hint: here the order of the books is relevant, as they are distinct.

- (9) Suppose the probability that an individual has a bad reaction from a flu vaccine is 0.001. Suppose 1000 individuals who are given the vaccine are sampled and asked about their reaction. Find the probability that of these more than 3 individuals have had a bad reaction: (a) using the Binomial distribution, (b) using the Poisson distribution as an approximation.

- (10) For problem (9), what are the mean and variance of the corresponding Binomial and Poisson random variables.