Not from textbook

$A = c(4.21, 4.13, \ldots, 4.27)$

$B = c(4.27, 4.38, \ldots, 4.41)$

$\mu_1 = \text{mean}(A) = 4.214$; $\mu_2 = \text{mean}(B) = 4.323$

$sd(A) = S_1 = 0.0683$

$sd(B) = S_2 = 0.0750$

99% CI $\Rightarrow \alpha = 0.01 \Rightarrow \alpha/2 = 0.005$

90% CI $\Rightarrow \alpha = 0.10 \Rightarrow \frac{\alpha}{2} = 0.05$

CI given by:

$$(\bar{X}_1 - \bar{X}_2) - t_{\alpha/2, n_p} \sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$\leq (\mu_1 - \mu_2) \leq (\bar{X}_1 - \bar{X}_2) + t_{\alpha/2, n_p} \sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

with $n_p = n_1 + n_2 - 2$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$np = n_1 + n_2 - 2 = 18$

Need $t_{\alpha/2, np}$ for $\frac{\alpha}{2} = 0.05$ and $\frac{\alpha}{2} = 0.005$

$\underbrace{t_{0.05}}, 18 = 1.734$ (from table)
"area in one tail"

using R, we can get this value via:

$qt(1-0.05, df=18) = t_{90}$

qt is the inverse CDF function of the t-distribution. We are looking for a value of the 95th %-ile of this distribution.

Similarly,

$t_{0.005, 18} = qt(1-0.005, 18) = 2.878$
$= t_{99}$

using these values, we can construct the two CIs (see R code) for $\mu_1 - \mu_2$:

$$E90 = t_{90} \cdot \text{sqrt}\left(sp2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)\right).$$

$$CI90 = c((m1-m2) - E90, (m1-m2)+E90))$$

$$= (-0.163, -0.055).$$

tighter interval
smaller confidence

similarly using t_{99}, we get the interval:

$$CI99 = (-0.199, -0.0195)$$

wider interval
higher confidence

These indicate that m_2 is likely larger than m_1 (that is both at 90% and 99% confidence levels, mean saturated fat content of brand 2 is greater than that of brand 1).

If we instead use the std normal distribution (which would be correct only if we could estimate σ_1, σ_2 with high confidence, which in this case we cannot), we get:

$$E_{90n} = z_{0.05} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

where we use $\sigma_1 \approx s_1$ and $\sigma_2 \approx s_2$.

$\Rightarrow (\bar{X}_1 - \bar{X}_2) \pm E_{90n}$ is 90% CI in this case. The posted R code gives us:

$z_{90} = \text{qnorm}(1-0.05) = 1.645 = z_{0.1/2}$

$z_{99} = \text{qnorm}(1-0.005) = 2.576 = z_{0.01/2}$

$CI90z = (-0.162, -0.0562)$

$CI99z = (-0.192, -0.0263)$

Notice that both of these intervals are fairly close to those obtained with the t-distribution and both support the same conclusion for the yogurt brands as above.