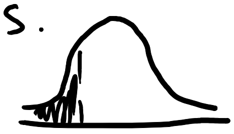
Ex) Suppose 100 tires lasted avg 21,500 miles with std deviation 1298 miles.



$-z_\alpha$

Test $H_0 : \mu = 22{,}000$ $\qquad \alpha = 0.05$ $\qquad$ Since n large $(n > 30)$

$H_1 : \mu < 22{,}000$ $\qquad \sigma \approx s$

$z_\alpha = 1.645 \Rightarrow$ Reject H_0 if $z \leq -1.645 = -z_\alpha$.

$$z_s = \frac{\bar{x} - \mu_0}{\sigma_0/\sqrt{n}} = \frac{21{,}500 - 22{,}000}{1298/\sqrt{100}} \approx -3.85$$

Since $z_s < z_\alpha$, we reject H_0 at $\alpha = 0.05$ confidence level.

<u>p-value characterization</u>

Previous method: select α prior to conducting test, keep or reject H_0 based on value of statistic.

p-value method:

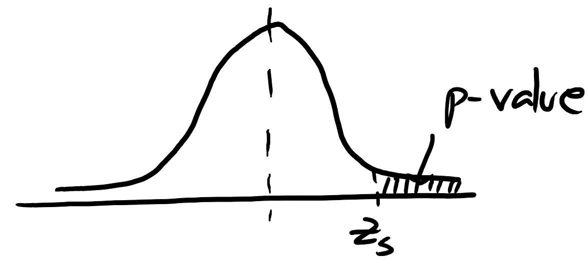
The <u>observed significance level</u>, or <u>p-value</u>, for a specified statistical test is the probability (assuming H_0 is true) of observing a value of the test statistic that is at least as contradictory to H_0 as the actual one computed from the sample data.

Ex) (city pipes) $n=50$, $\bar{x}=2460$, $\sigma=200$

$H_0: \mu = 2400$

$H_1: \mu > 2400$

$$z_s = \frac{2460-2400}{200/\sqrt{50}} \approx 2.12$$



$$\text{p-value} = P(z \geq 2.12)$$

$$= 0.5 - .4830 = .0170$$

$\Rightarrow$ The probability of observing a z value as large as 2.12 is only 0.0170, if in fact the true value of μ is 2400.

If you are inclined to select $\alpha = .05$ for this test, then you would reject the null H_0 because p-value $= .017 < 0.05$. In contrast, if you were to choose $\alpha = .01$ for this test, you would not reject H_0.

Thus, the choice of α-value is left to you.

Ex) Research neurologist is testing the effects of a drug on response time by injecting 100 rats with a dose of the drug.

mean response time for rats w/o drug = 1.2 s

mean response time " " w/ drug = 1.05 sec

s = .5 sec.

H_0: $\mu = 1.2$

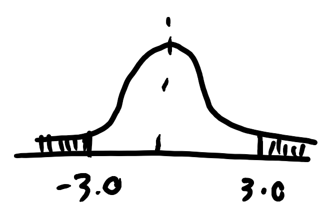
H_1: $\mu \neq 1.2$ $\quad (\mu < 1.2$ or $\mu > 1.2)$

$$z_s = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} \approx \frac{\bar{x} - 1.2}{s/\sqrt{n}} \quad \text{(since } n > 30\text{)}$$

$s/\sqrt{n} \approx \sigma/\sqrt{n}$

$$= \frac{1.05 - 1.2}{.5/\sqrt{100}} \approx -3.0$$

$\Rightarrow$ The sample mean 1.05 is approx 3 std deviations below the H_0 mean of 1.2 in the sampling distribution of $\bar{x}$.



$$\text{p-value} = P(z < -3.0 \text{ or } z > 3.0)$$
$$= 2[0.5 - P(0 < z < 3.0)]$$
$$= 2[0.5 - .4987] = .0013 \times 2 = 0.0026$$

We can interpret this p-value as a strong indication that the mean reaction time of drug injected rats differs from the control mean ($\mu \neq 1.2$) since we would observe a test statistic this extreme or more extreme only 26 in 10,000 times if the drug injected mean were equal to the control mean ($\mu = 1.2$).

How to decide whether to reject H_0 with p-values

1. Choose the max value of α that you are willing to tolerate. (max p(type I error))

2. Reject H_0 if the p-value $< \alpha$. Otherwise, do not reject H_0.

Ex) An experiment is performed to determine whether the avg mercury content in 1kg packs of wild salmon exceeds that of another kind by .20mg.

$n_1 = 30$, $\bar{x}_1 = 2.61$ mg, $s_1 = 0.12$ mg

$n_2 = 25$, $\bar{x}_2 = 2.38$mg, $s_2 = 0.14$ mg

use $\alpha = 0.05$

$H_0: \mu_1 - \mu_2 = 0.20$

$H_1: \mu_1 - \mu_2 \neq 0.20$

$d = 0.20$

$$t_s = \frac{\bar{x}_1 - \bar{x}_2 - d}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad ; \quad s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$$

pooled variance

"weighted sum"

Reject H_0 if: $|t_s| \geq t_{\alpha/2, n_1+n_2-2}$

$$= t_{0.025, 30+25-2}$$

$$= t_{0.025, 53} \approx 2.0$$

$$t_s = \frac{(2.61-2.38)-0.20}{\sqrt{0.017\left(\frac{1}{30}+\frac{1}{25}\right)}} \approx 0.85$$

The null hypothesis H_0 may not be rejected at $\alpha = 0.05$.
The difference $2.61 - 2.38 = 0.23$ is not significantly different from 0.2. The difference may well be attributed to chance.

Ex) Let P be a normal population with variance $\sigma^2 = 0.25$ and unknown mean μ.

Suppose: $H_0: \mu = 1$ vs $H_1: \mu = 2$

Find α, β, power of test.

Let $\bar{x}$ denote the observed value of the mean of a random sample of size 10 from the population.

Decision rule: Reject H_0 when $\bar{x} > 1.40$.

$\alpha = P(\text{reject } H_0 \text{ when } H_0 \text{ is true})$

$= P(\bar{x} > 1.40 \mid \mu = \mu_0 = 1)$

$$= P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{1.40-1}{.5/\sqrt{10}}\right) = P(Z > 2.53) \approx 0.01$$

b/c pop. is normal.

$\beta = P(\text{accept } H_0 \text{ when } H_0 \text{ is false})$ $\quad H_1$ true $\Rightarrow \mu = 2$

$= P(\text{accept } H_0 \text{ when } H_1 \text{ is true})$

$$= P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < \frac{1.40-2}{.5/\sqrt{10}}\right) = P(Z < -3.79) \approx 0.001$$

Power of test $= P(\text{reject } H_0 \text{ when } H_1 \text{ is true})$

$= 1 - P(\text{accept } H_0 \text{ when } H_1 \text{ is true}) = 1 - P(\text{type II error})$

$= 1 - \beta \approx 1 - 0.001 = 0.999$

Ex) Suppose a sample of 1600 tires is obtained at random from one manufacturer with p-defective = 0.08. What is the probability that in this sample not more than 150 tires will be defective?

B : r.v. counting # defective tires

$$P(B \le 150) = P(B < 150.5) \approx P\left(Z < \frac{150.5 - \overset{np_0}{128}}{\underbrace{10.85}_{\sqrt{np_0(1-p_0)}}}\right)$$

$= P(Z < 2.07) \approx 0.98$

Ex) If 4 of 20 patients suffered side effects from a new medication, test the null hypothesis that $\theta = 0.50$ against $\theta \neq 0.50$ at 0.05 level.

$H_0: \theta = 0.50$

$H_1: \theta \neq 0.50$

X: binom r.v. counting # patients with bad reaction

Suppose approx normal population.

$$z_s = \frac{x - n\theta_0}{\sqrt{n\theta_0(1-\theta_0)}} = \frac{4 - 10}{\sqrt{20(0.5)(0.5)}} \approx -2.68$$

$z_\alpha = 1.645 \Rightarrow$ Since $z_s < -z_\alpha$ we reject H_0 at $\alpha = 0.05$ level.

What is the p-value? $P(X \leq 4 \text{ or } X \geq 16)$

Calculate $P(X \leq 4) = 0.0059$ or $P(X \geq 16)$

use R: pbinom(4, 20, 0.5) (CDF)

p-value $= 2 \times (0.0059) = 0.0118$

since p-value < 0.05, the null H_0 must be rejected at given α.