Review of basic probability

Sample space : set S consisting of all possible outcomes of a random experiment.

Ex) Toss a die. $S = \{1,2,3,4,5,6\}$

Event : Subset of the sample space S. An event E consisting of a single point of S is called a simple event.

Mutually exclusive, disjoint events

Are events A, B for which $A \cap B = \phi$.

Ex) Toss a coin twice. Let A be the even that "at least one head occurs".
Let B be the event "the second toss results in a tail".

Find $P(A)$, $P(B)$, $P(A \cup B)$, $P(A \cap B)$
$P(A')$, $P(A-B) = P(A \cap B')$.

$S = \{HH, TT, HT, TH\}$

$A = \{HT, TH, HH\} \Rightarrow P(A) = \frac{3}{4}$

$B = \{HT, TT\} \Rightarrow P(B) = \frac{2}{4} = \frac{1}{2}$

$A \cup B = \{HT, TH, HH, TT\} = S$

$\Rightarrow P(A \cup B) = 1.$

$A \cap B = \{HT\} \Rightarrow P(A \cap B) = \frac{1}{4}$

$A' = \{TT\} \Rightarrow P(A') = \frac{1}{4}$

$A - B = A \cap B' =$

$= \{HT, TH, HH\} \cap \{HH, TH\}$

$= \{HH, TH\} \Rightarrow P(A-B) = \frac{1}{2}.$

Rules

$0 \leq P(A) \leq 1$

$P(S) = 1$

$P(\phi) = 0$

$P(A') = 1 - P(A)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

when $A \cap B = \phi \Rightarrow P(A \cup B) = P(A) + P(B)$

$P(A \text{ exclusive or } B) = P(A) + P(B) - 2P(A \cap B)$

Ex) Suppose in a community of 400 adults, 300 bike or swim or do both, 160 swim, and 120 swim and bike.

What is the probability that an adult selected at random from this community bikes?

A: The person swims

B: The person bikes

$$\Rightarrow P(A \cup B) = \frac{300}{400} \; ; \; P(A) = \frac{160}{400}$$

$$P(A \cap B) = \frac{120}{400}$$

$$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(B) = P(A \cup B) - P(A) + P(A \cap B)$$

$$= \frac{300}{400} - \frac{160}{400} + \frac{120}{400} = \frac{260}{400} = 0.65$$

Independent events

A and B are independent if the occurrence of A does not influence the occurrence of B.

$$P(A \cap B) = P(A)\,P(B)$$

Ex) In tossing four fair dice, what is the probability of at least one 3?

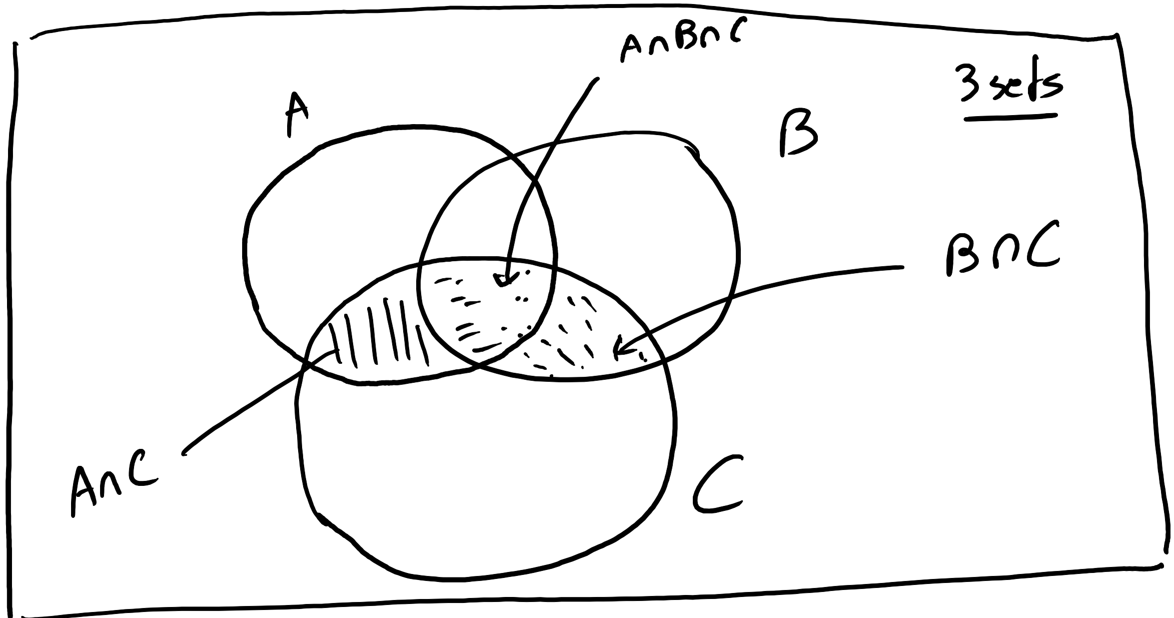
A: get at least one 3

A': get no 3 in 4 tosses

C: no 3 in 1 toss $\Rightarrow P(C) = \frac{5}{6}$

$P(A') = \left(\frac{5}{6}\right)^4$ each of the four tosses is independent.

$P(A) = 1 - P(A') = 1 - \left(\frac{5}{6}\right)^4.$



$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Ex) Suppose that 25% of the population of a city read newspaper A, 20% read newspaper B, 13% read C, 10% read both A and B, 8% read both A and C, 5% read B and C, 4% read all three. If a person from this city is selected at random, what is the probability that he or she does not read any of these newspapers?

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$
$$- P(A \cap B) - P(A \cap C) - P(B \cap C)$$
$$+ P(A \cap B \cap C)$$

$$= 0.25 + 0.20 + 0.13 - 0.10 - 0.08 - 0.05$$
$$+ 0.04 = 0.39$$

$P(\text{reads no newspapers}) = 1 - P(A \cup B \cup C)$
$$= 1 - 0.39 = 0.61$$

Ex) (Birthday problem) What is the probability that at least two students in a class of size n have the same birthday? (Assume year = 365 days).

There are 365 possibilities for the birthdays of each of the n students.

$\Rightarrow$ sample space has 365^n points.

In order for birthdays not to coincide:

365 choices	364 choices	....	$365-(n-1)$ choices
person 1	person 2		person n

$$P(\text{no shared bday}) = \frac{365 \times 364 \times \ldots \times [365-(n-1)]}{365^n}$$

$$P(\overset{\text{at least one}}{\text{shared}}\ \text{bday}) = 1 - P(\text{no shared bday})$$

for $n=30$,

$P(\text{at least one shared}) = 0.706$

for $n=50$,

$P(\ldots) = 0.970$ (very high!)